Designing Complex Dynamics with Memory: Elementary Cellular Automata Case

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Embryo Physics Course
http://embryogenesisexplained.com/
video conference via *Second Life*, February 19, 2014

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This presentation is basically the next paper

available freely in PDF from the next link: http://eprints.uwe.ac.uk/21980/

Motivation

Since their inception at `Macy conferences' in later 1940s  complex systems remain the most controversial topic of inter-disciplinary sciences. The term `complex system' is the most vague and liberally used scientific term. Using elementary cellular automata (ECA), and exploiting the CA classification, we demonstrate elusiveness of `complexity' by shifting space-time dynamics of the automata from simple to complex by enriching cells with `memory'. This way, we can transform any ECA class to another ECA class -- without changing skeleton of cell-state transition function --- and vice versa by just selecting a right kind of memory. A systematic analysis display that memory helps `discover' hidden information and behaviour on trivial -- uniform, periodic, and non-trivial -- chaotic, complex -- dynamical systems.

Nature vs Complex Systems
we have two interesting point of view ...

FINITE NATURE

EDWARD FREDKIN
DEPARTMENT OF PHYSICS
BOSTON UNIVERSITY
BOSTON, MA, 02215, USA

ABSTRACT
1992

A fundamental question about time, space and the inhabitants thereof is "Are things smooth or grainy?" Some things are obviously grainy (matter, charge, angular momentum); for other things (space, time, momentum, energy) the answers are not clear. Finite Nature is the assumption that, at some scale, space and time are discrete and that the number of possible states of every finite volume of space-time is finite. In other words Finite Nature assumes that there is no thing that is smooth or continuous and that there are no infinitesimals. If finite nature is true, then there are certain consequences that are independent of the scale.

Stephen Wolfram's
A NEW KIND OF SCIENCE

The Crucial Experiment

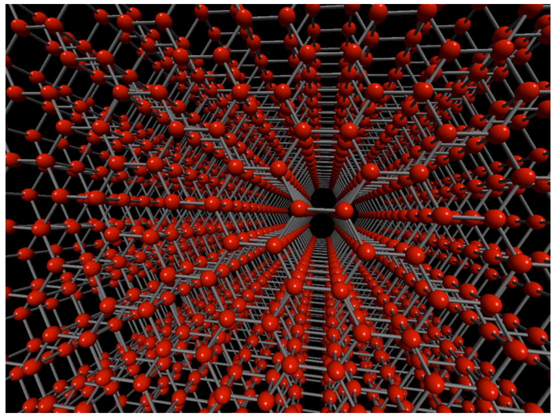
2002

How Do Simple Programs Behave?

New directions in science have typically been initiated by certain central observations or experiments. And for the kind of science that I describe in this book these concerned the behavior of simple programs.

both theories relate the cellular automata theory

Cellular automata



Cellular automata (CA) are discrete dynamical systems evolving on an infinite regular lattice.

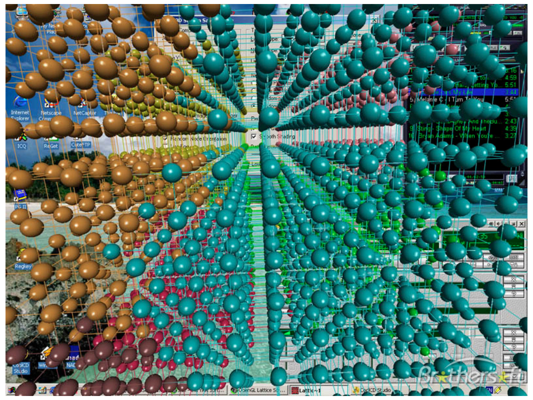
A *CA* is a 4-tuple $A = <\Sigma, \mu, \phi, c_0>$ evolving in d-dimensional lattice, where $d \in Z^+$. Such that:

Σ represents the finite *alphabet*

μ is the *local connection*, where, $\mu = \{x_{0,1,\ldots,n-1:d} \mid x \in \Sigma\}$, therefore, μ is a *neighbourhood*

ϕ is the *local function*, such that, $\phi : \Sigma^\mu \rightarrow \Sigma$

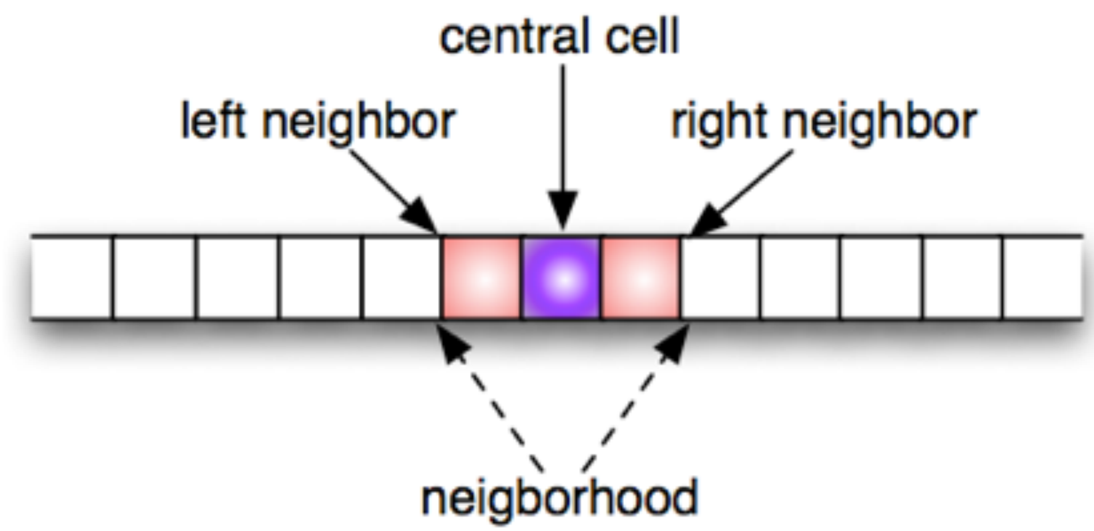
c_0 is the *initial condition*, such that, $c_0 \in \Sigma^Z$

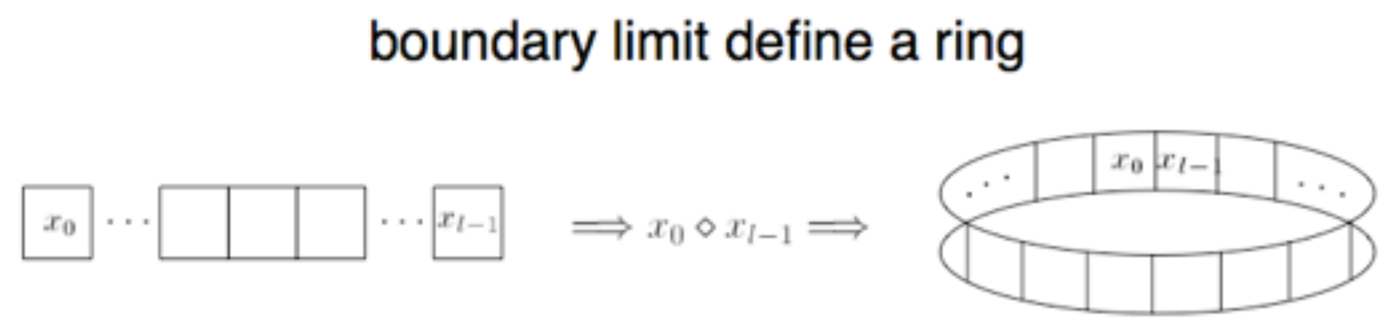


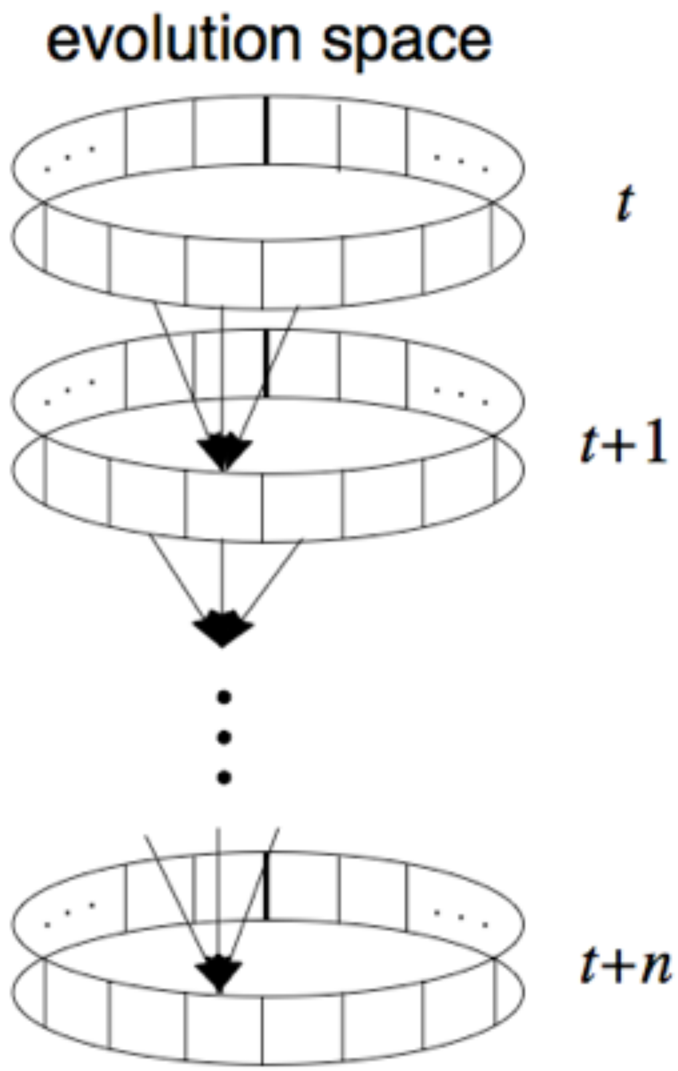
Also, the local function induces a *global transition* between configurations:

$$\Phi_\phi : \Sigma^Z \rightarrow \Sigma^Z.$$

CA dynamics in one dimension







Elemental CA (ECA) is defined as follows:

$\Sigma = \{0,1\}$

$\mu = (x_{+1}, x_0, x_{-1})$ such that $x \in \Sigma$

$\phi : \Sigma^3 \rightarrow \Sigma$

$\mu = \{c_0 \mid x \in \Sigma\}$ the initial condition is the first ring with $t = 0$

Wolfram classes in CA

Wolfram defines his classification in simple rules [Wolfram, 1986], known as ECA. Also, this classification is extended to *any* dimension.

A CA is **class I**, if there is a stable state $x_i \in \Sigma$, such that all finite configurations evolve to the ***homogeneous configuration***.

A CA is **class II**, if there is a stable state $x_i \in \Sigma$, such that any finite configuration ***become periodic***.

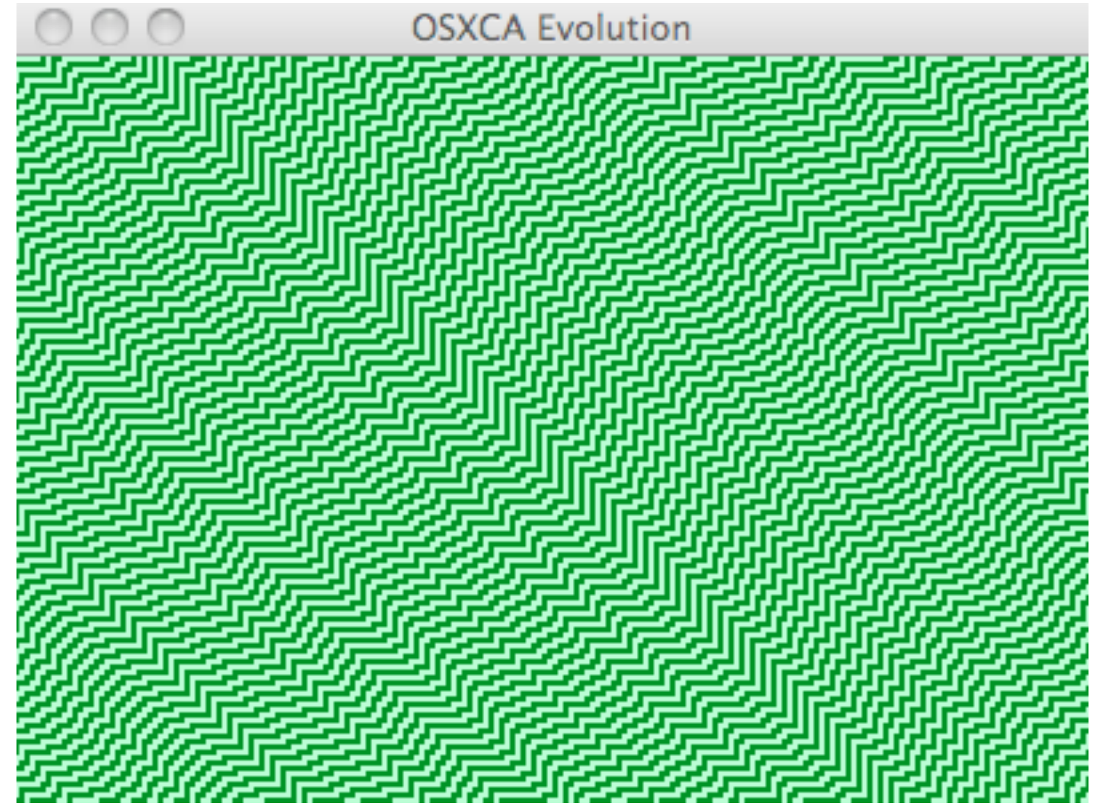
A CA is **class III**, if there is a stable state, such that for some pair of finite configurations c_i and c_j with the stable state, is decidable if c_i evolve to c_j, such that any configuration ***become chaotic***.

Class IV includes all previous CA, also ***called complex***. [Culik II & Yu, 1988]

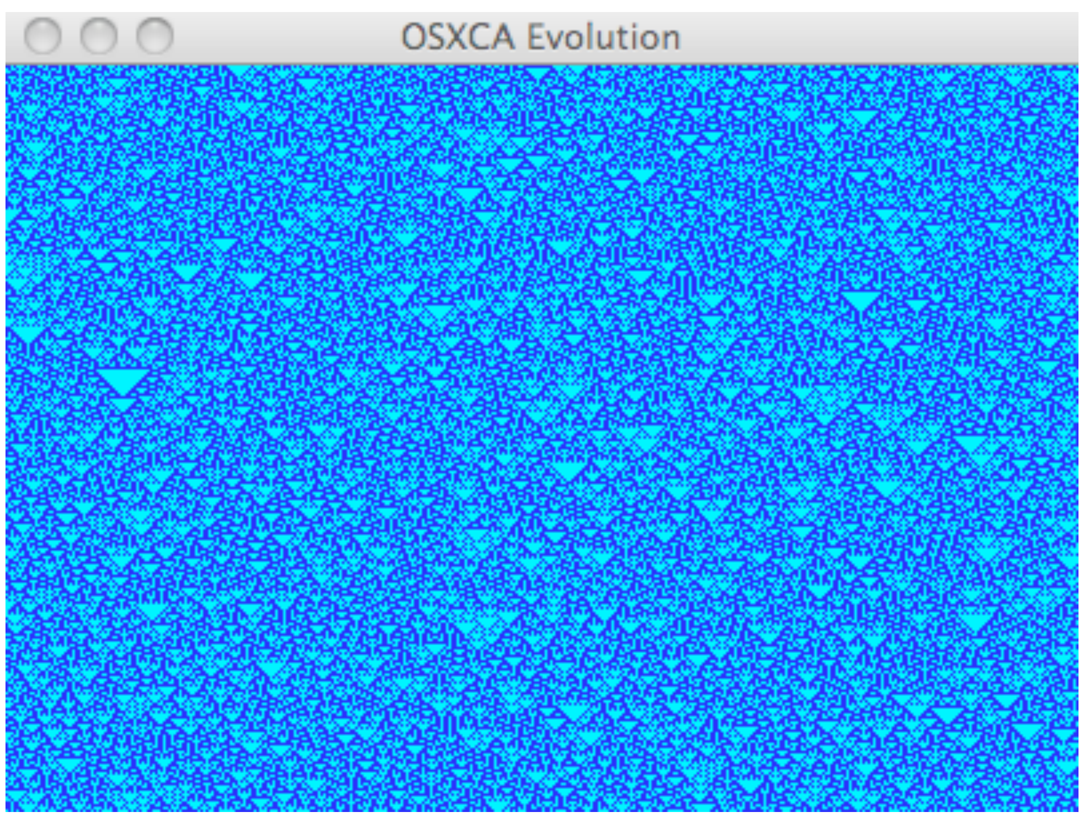
Stephen Wolfram, ***Cellular Automata and Complexity***, Addison-Wesley Publishing Company, 1994.
Karel Culik II and Sheng Yu, "**Undecidability of CA Classification Schemes**," *Complex Systems* 2, 177-190, 1988.

CA classes in one dimension

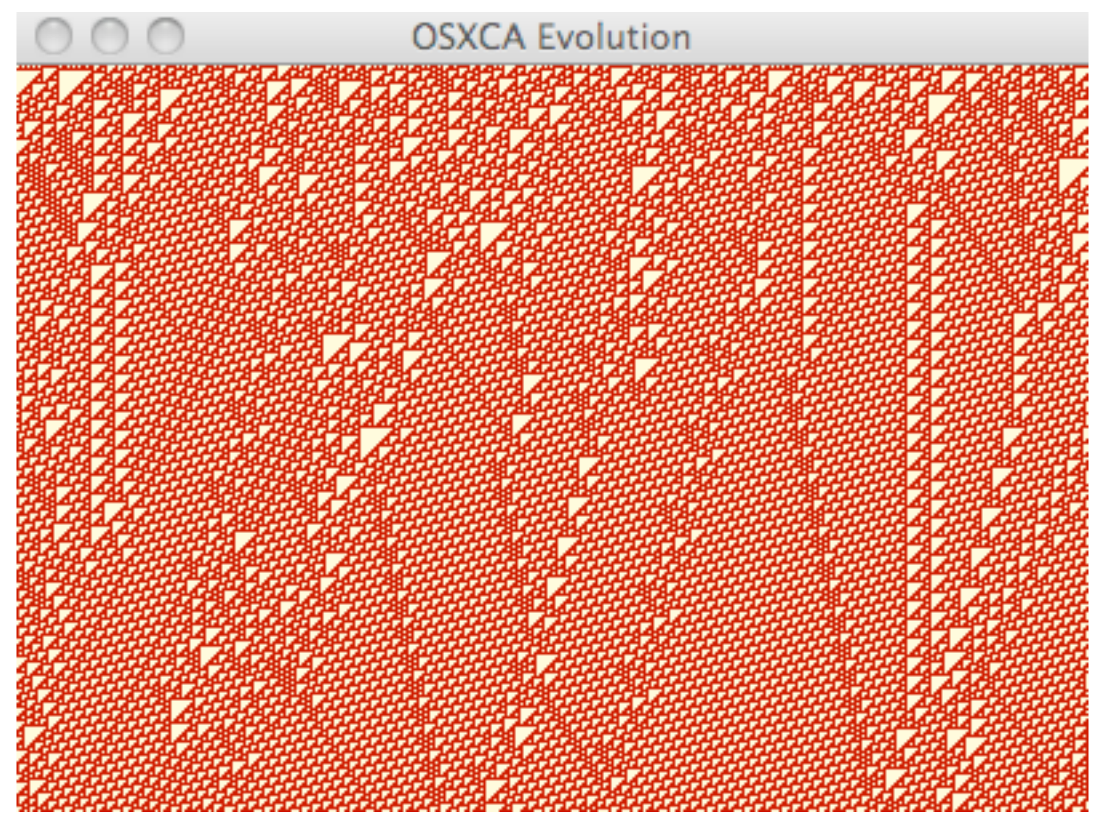
class I: uniform



class II: periodic



class III: chaotic



class IV: complex

Mean field approximation: *fixed points*

Mean field theory [Gutowitz, 1984] is a proven technique for discovering statistical properties of CA without analyzing evolution spaces of individual rules. In this way, it was proposed to explain Wolfram's classes by probability theory, resulting in a classification based on mean field theory curve [McIntosh, 1990]:

class I: monotonic, entirely on one side of diagonal;
class II: horizontal tangency, never reaches diagonal;
class IV: horizontal plus diagonal tangency, no crossing;
class III: no tangencies, curve crosses diagonal.

Thus for one dimension we have:

$$p_{t+1} = \Sigma_{j=0}^{k^{2r+1}-1} \phi_j(X) p_t^{v} (1-p_t)^{n-v}$$

such that *j* is a number of relations from their neighbourhoods and *X* the combination of cells $x_{i-r},\ldots,x_i,\ldots,x_{i+r}$. *n* represents the number of cells in neighbourhood, *v* indicates how often state one occurs in Moore's neighbourhood, *n*-*v* shows how often state zero occurs in the neighbourhood, p_t is a probability of cell being in state one, q_t is a probability of cell being in state zero (therefore q=1-p).

Howard A. Gutowitz, "**Mean Field vs. Wolfram Classification of Cellular Automata**," http://tuvalu.santafe.edu/~hag/mfw/mfw.html, 1989.
Harold V. McIntosh, "**Wolfram's Class IV and a Good Life**," *Physica D* 45, 105-121, 1990.

Field of basin attractors: *cycles*

Generally a basin could classifier CA with chaotic or complex behavior following also previous results on attractors [Wuensche, 1992-99].

class I: very short transients, mainly point attractors (but possibly also point attractors) (very ordered dynamics) very high in-degree, very high leaf density (ordered dynamics);

class II: very short transients, mainly short periodic attractors (but also point attractors), high in-degree, very high leaf density;

class III: very long transients, very long periodic attractors low in-degree, low leaf density (chaotic dynamics);

class IV: moderate transients, moderate length periodic attractors moderate in-degree, moderate very leaf density (possibly complex dynamics).

Andrew Wuensche, “**Classifying Cellular Automata Automatically**,” *Complexity* 4(3), 47-66, 1999.
Harold V. McIntosh, ”**Ancestors: Commentaries on The Global Dynamics of Cellular Automata** by Andrew Wuensche and Mike Lesser (Addison-Wesley, 1992),” *Workpaper,* Universidad Autónoma de Puebla, Puebla, México, 1993.

Elemental cellular automata with memory

Conventional CA are ahistoric (memoryless): i.e., the new state of a cell depends on the neighbourhood configuration solely at the preceding time step of ϕ. CA with *memory* can be considered as an extension of the standard framework of CA where every cell x_i is allowed to remember some period of its previous evolution [Alonso-Sanz, 2009].

Thus to implement a memory we design a memory function Φ, as follow:

$$\Phi(x^{t-\tau}_i, \ldots, x^{t-1}_i, x^t_i) \rightarrow s_i$$

such that $\tau < t$ determines the degree of memory backwards and each cell $s_i \in \Sigma$ being a state function of the series of states of the cell x_i with memory up to time-step. Finally to execute the evolution we apply the original rule as follows:

$$\phi(\ldots, s^t_{i-1}, s^t_i, s^t_{i+1}, \ldots) \rightarrow x^{t+1}_i.$$

Thus in CA with memory, while the mapping ϕ remains unaltered, historic memory of all past iterations is retained by featuring each cell as a summary of its past states from Φ. Therefore cells *canalize* memory to the map ϕ.

Ramon Alonso-Sanz, ***Cellular Automata with Memory***, Old City Publishing, 2009.

Elemental cellular automata with memory

Firstly we should consider a kind of memory, in this case the majority memory Φ_{maj} and then a value for τ. This value represent the number of cells backward to consider in the memory. Therefore a way to represent functions with memory and one ECA associated is proposed as follow:

$$\Phi_{CAm:\tau}$$

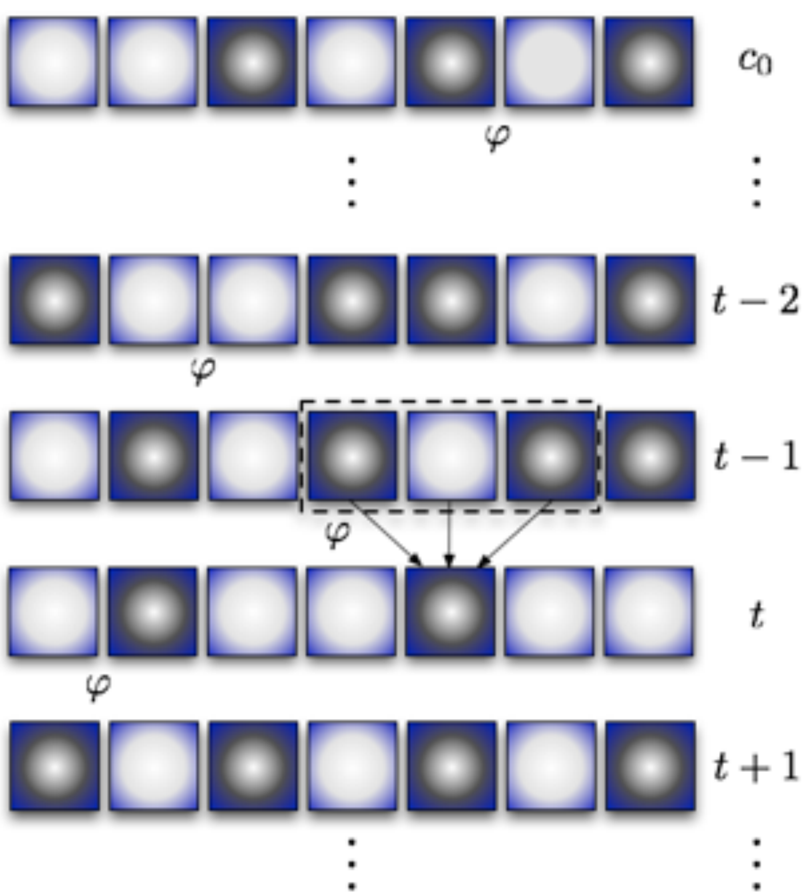
such that *CA* represents the decimal notation of an specific ECA and *m* a kind of memory given. This way the majority memory working in ECA rule 126 checking tree cells on its history is denoted simply as $\Phi_{R126maj:3}$.

Implementing the majority memory Φ_{maj} we can select some ECA and experimentally look what is the effect.

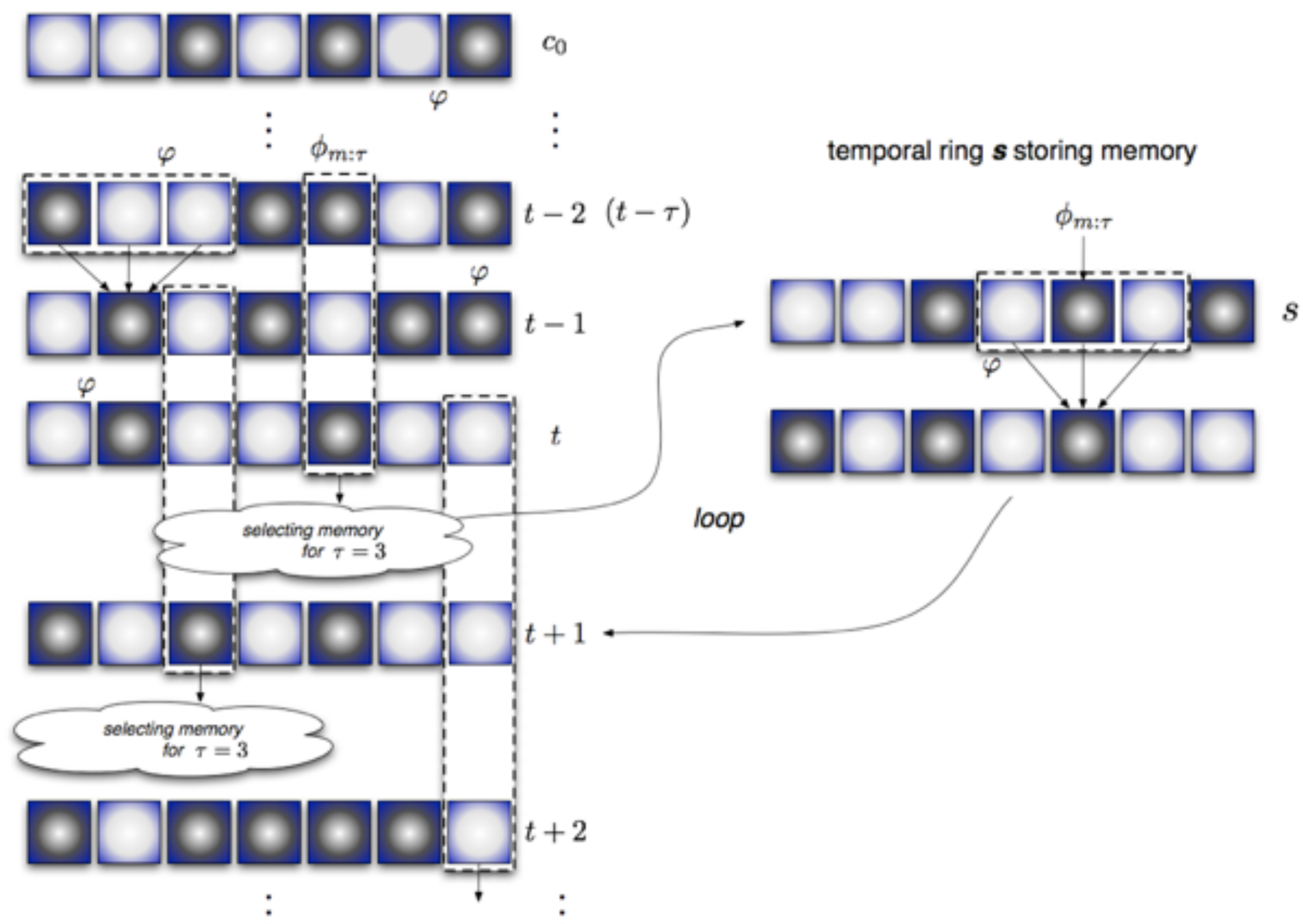
Ramon Alonso-Sanz, ***Cellular Automata with Memory***, World Scientific Series on Nonlinear Science, Series A, 2011.

Elementary Cellular Automata (ECA ahistoric)
Elementary Cellular Automata with Memory (ECAM historic)

conventional
CA evolution



historic
CA evolution



MEMORY: depend on the state and history of the system

We have demonstrated that: chaotic ECAM rule 30 has complex dynamics

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Complex Systems

Volume 18, Issue 3

Complex Dynamics Emerging in Rule 30 with Majority Memory

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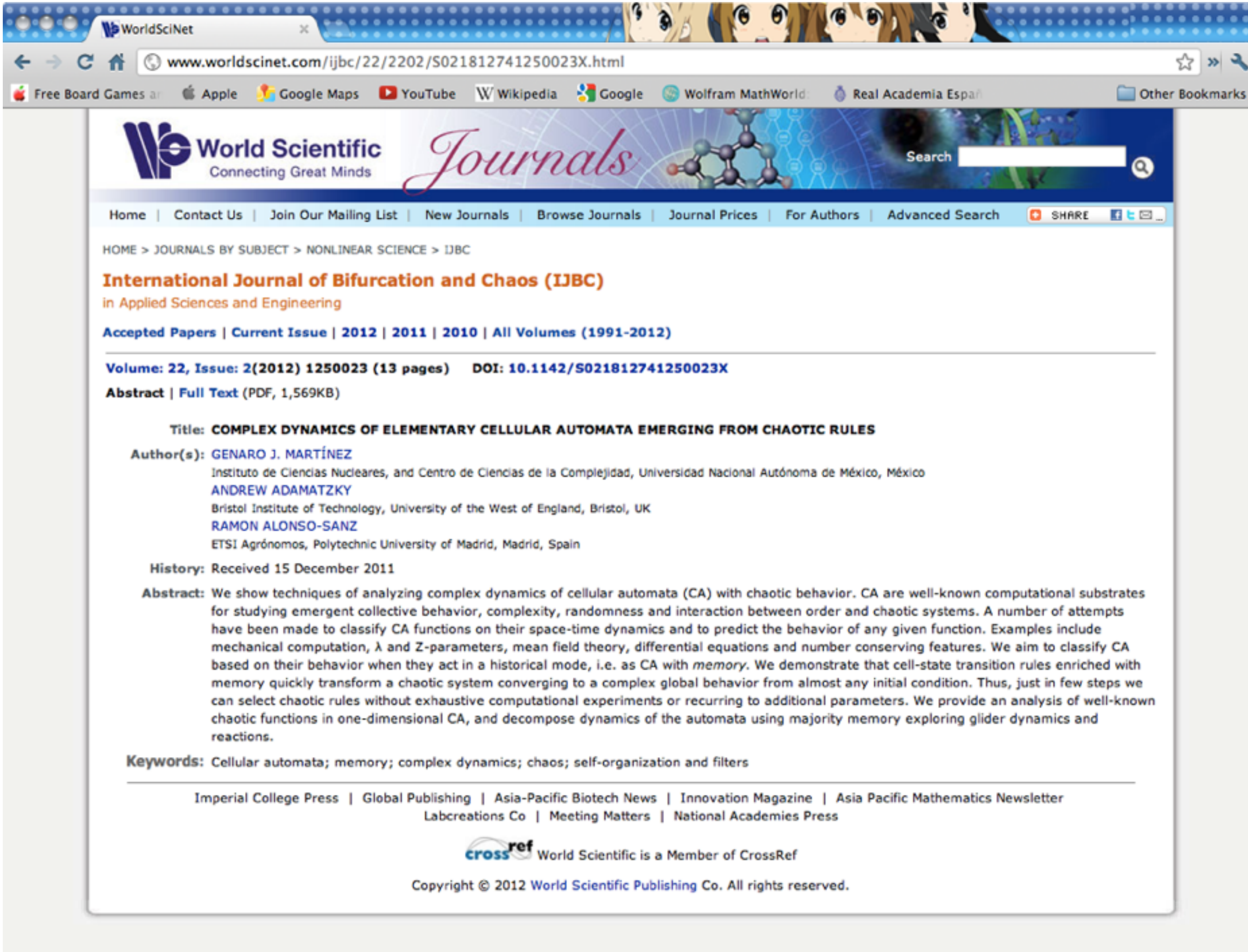
Abstract

In cellular automata (CAs) with memory, the unchanged maps of conventional CAs are applied to cells endowed with memory of their past states in some specified interval. We implement the rule 30 automaton and show that by using the majority memory function we can transform the quasi-chaotic dynamics of classical rule 30 into domains of traveling structures with predictable behavior. We analyze morphological complexity of the automata and classify glider dynamics (particle, self-localizations) in the memory-enriched rule 30. Formal ways of encoding and classifying glider dynamics using de Bruijn diagrams, soliton reactions, and quasi-chemical representations are provided.

SHARE

We have demonstrated that: chaotic ECAM rule 126 has complex dynamics

We have demonstrated that: chaotic ECAM rule 101 has complex dynamics



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Accepted Papers | Current Issue | 2012 | 2011 | 2010 | All Volumes (1991-2012)

Volume: 22, Issue: 2(2012) 1250023 (13 pages) DOI: 10.1142/S021812741250023X

Abstract | **Full Text** (PDF, 1,569KB)

Title: COMPLEX DYNAMICS OF ELEMENTARY CELLULAR AUTOMATA EMERGING FROM CHAOTIC RULES

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ETSI Agrónomos, Polytechnic University of Madrid, Madrid, Spain

History: Received 15 December 2011

Abstract: We show techniques of analyzing complex dynamics of cellular automata (CA) with chaotic behavior. CA are well-known computational substrates for studying emergent collective behavior, complexity, randomness and interaction between order and chaotic systems. A number of attempts have been made to classify CA functions on their space-time dynamics and to predict the behavior of any given function. Examples include mechanical computation, λ and Z-parameters, mean field theory, differential equations and number conserving features. We aim to classify CA based on their behavior when they act in a historical mode, i.e. as CA with *memory*. We demonstrate that cell-state transition rules enriched with memory quickly transform a chaotic system converging to a complex global behavior from almost any initial condition. Thus, just in few steps we can select chaotic rules without exhaustive computational experiments or recurring to additional parameters. We provide an analysis of well-known chaotic functions in one-dimensional CA, and decompose dynamics of the automata using majority memory exploring glider dynamics and reactions.

Keywords: Cellular automata; memory; complex dynamics; chaos; self-organization and filters

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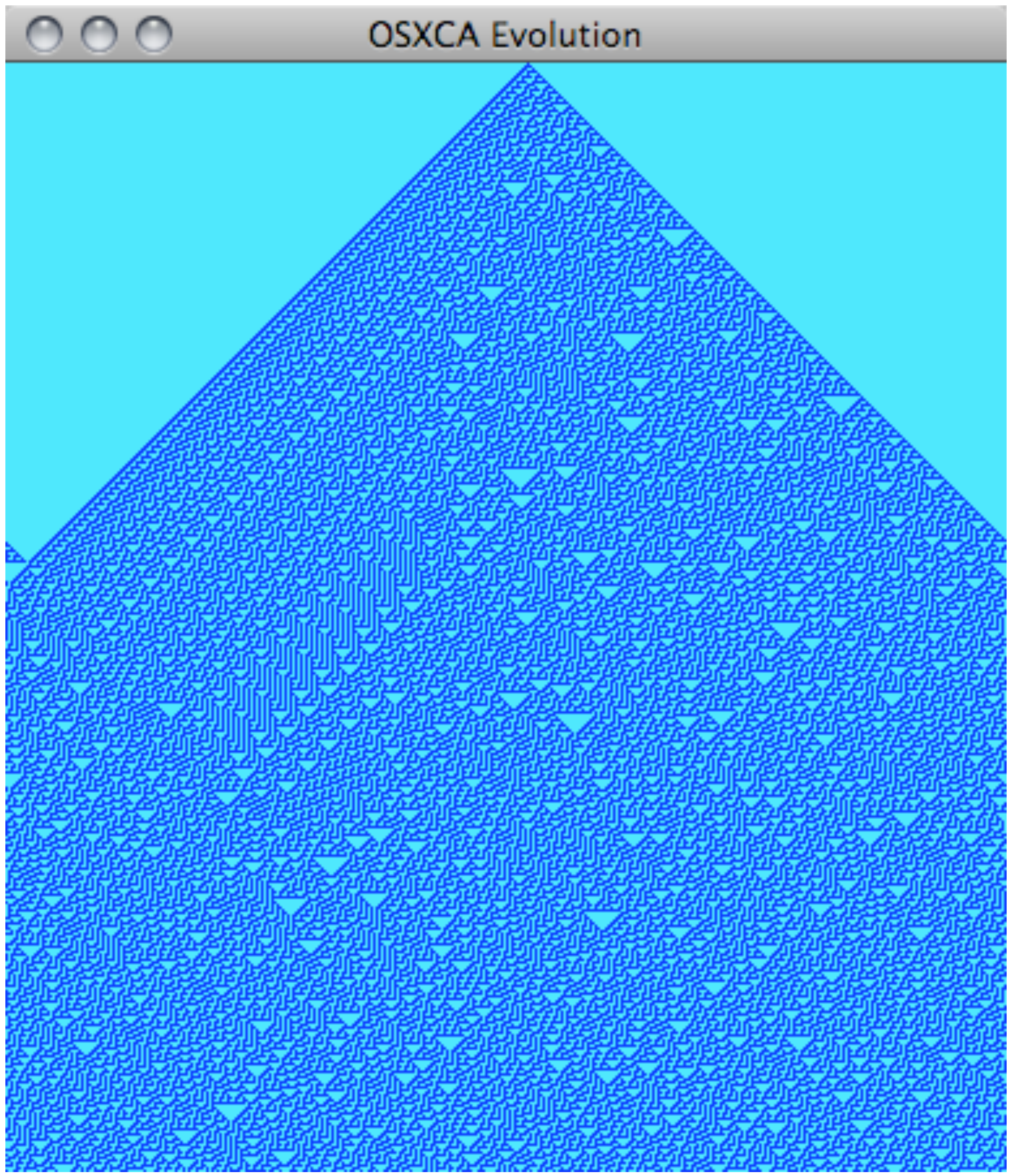
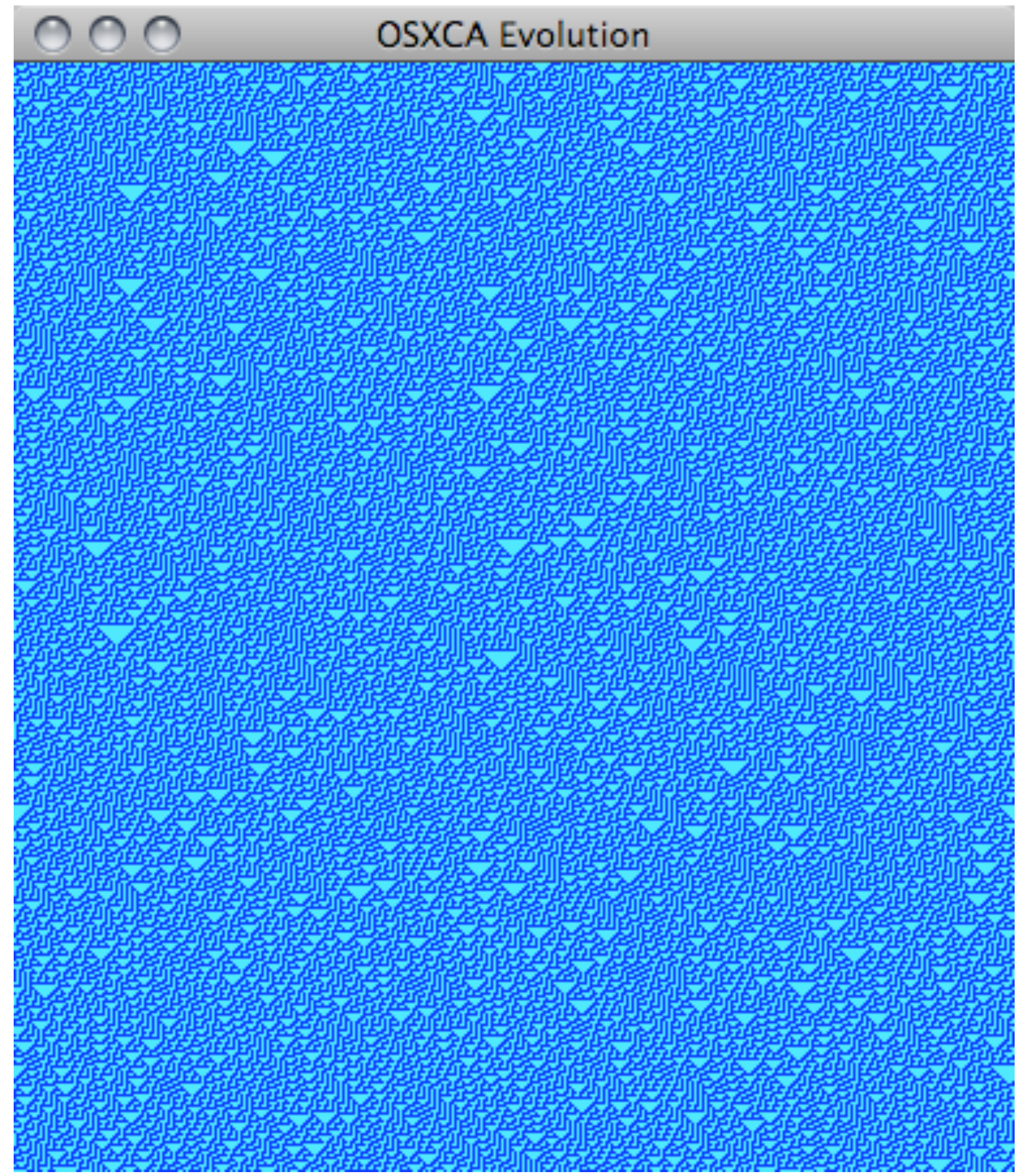
Two cases of study

ECA Rule 30 and Rule 126

Chaotic ECA rule 30: evolution space

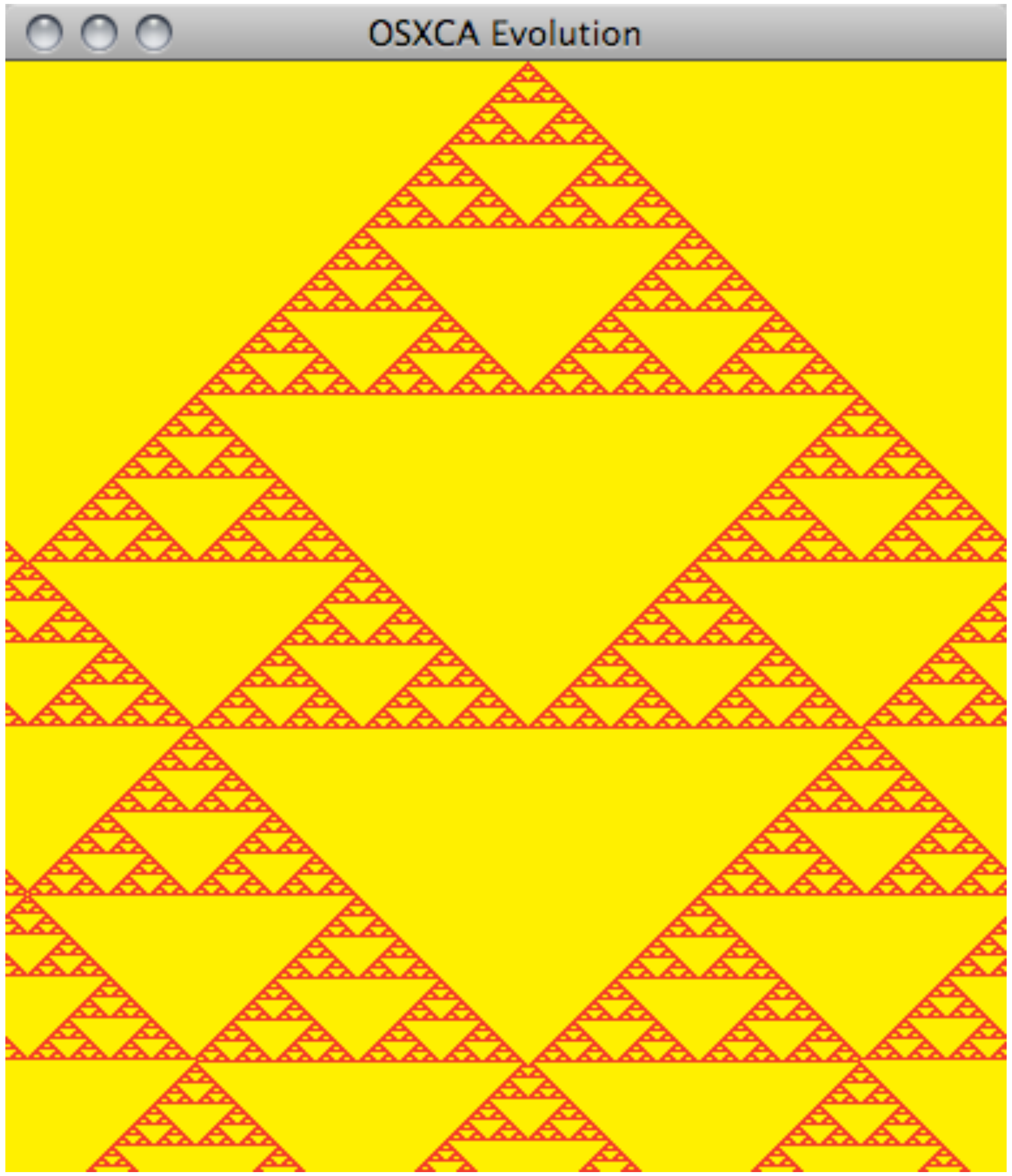


one cell in state 1

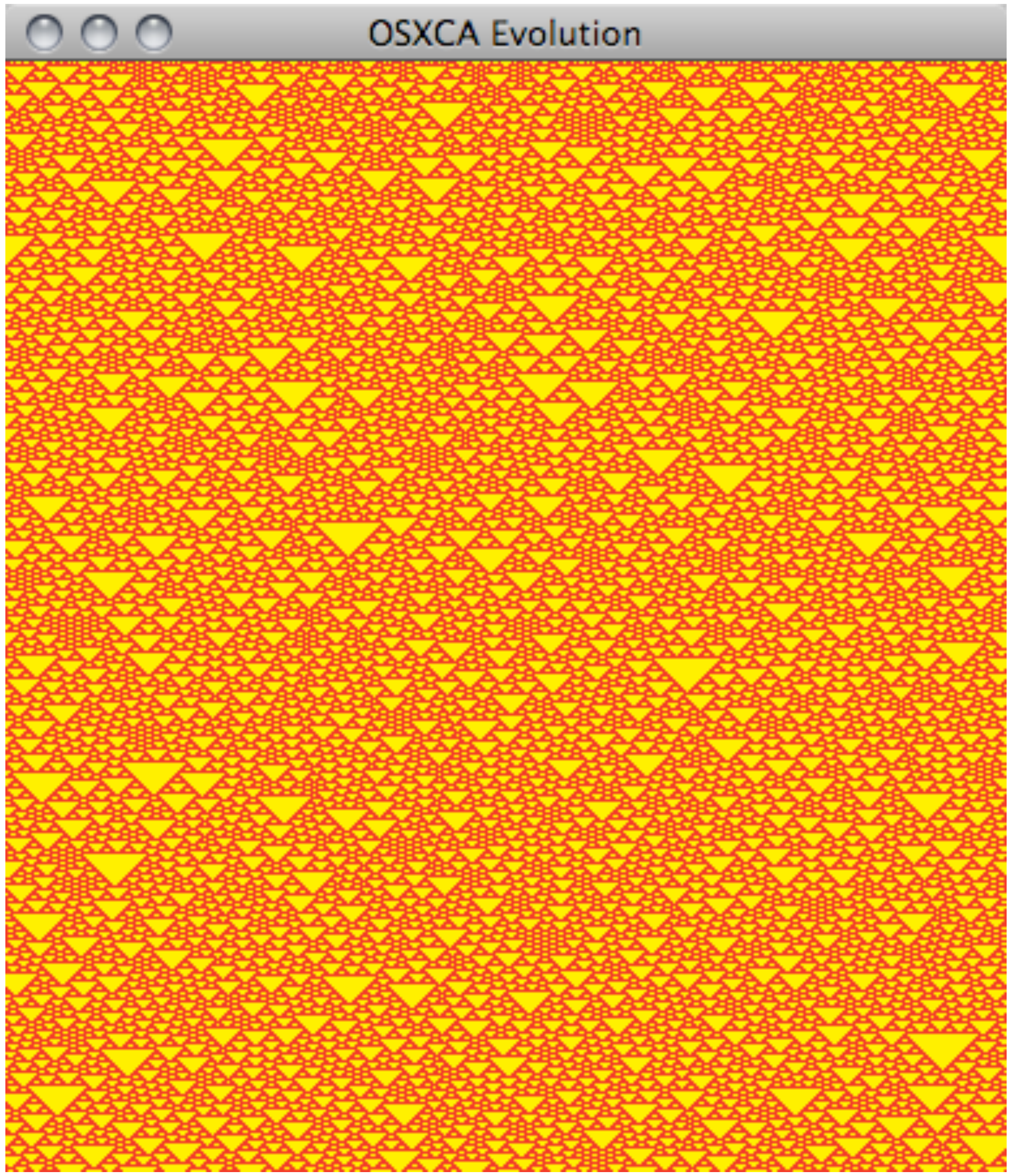


random initial condition 50%

Chaotic ECA rule 126: evolution space

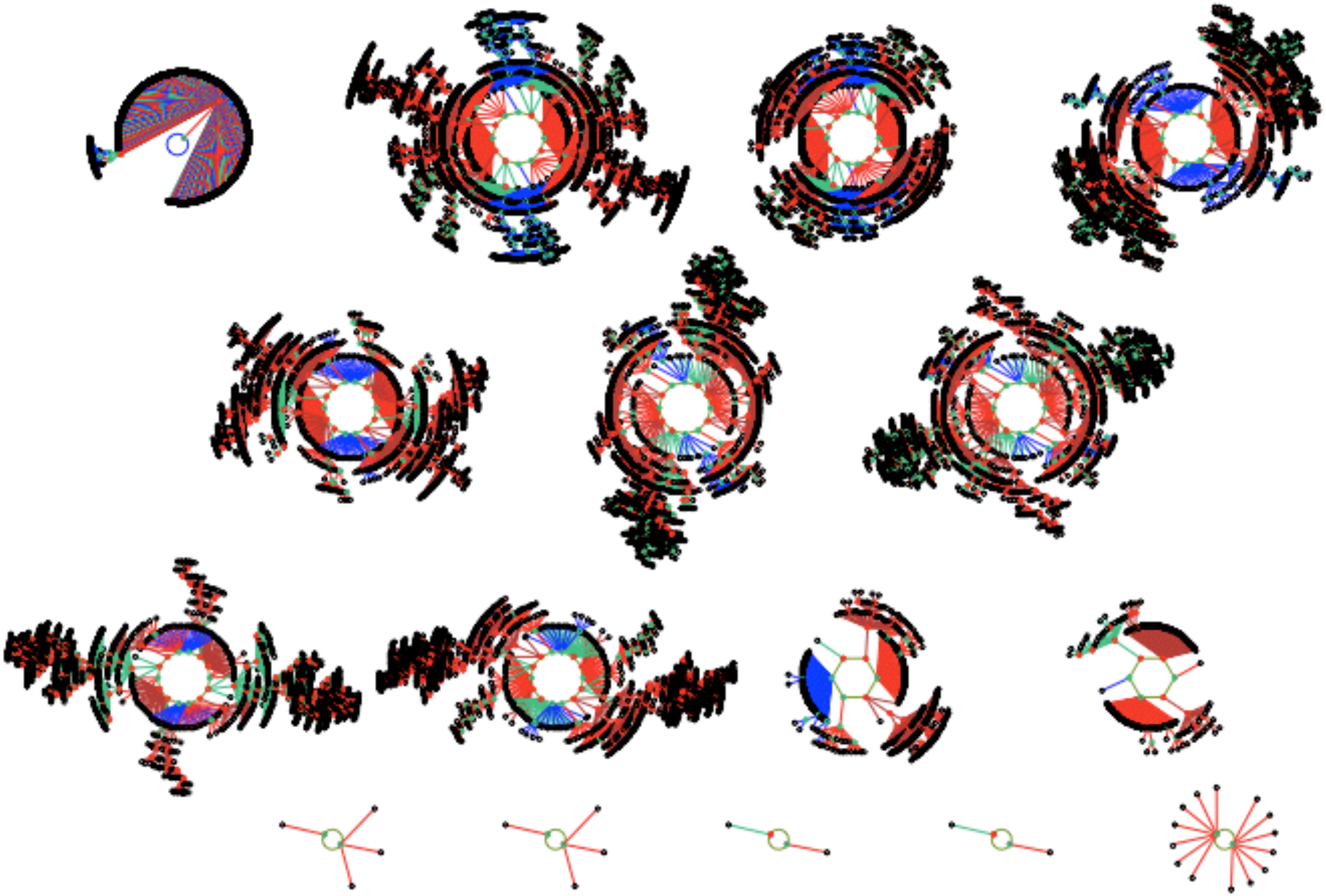


one cell in state 1



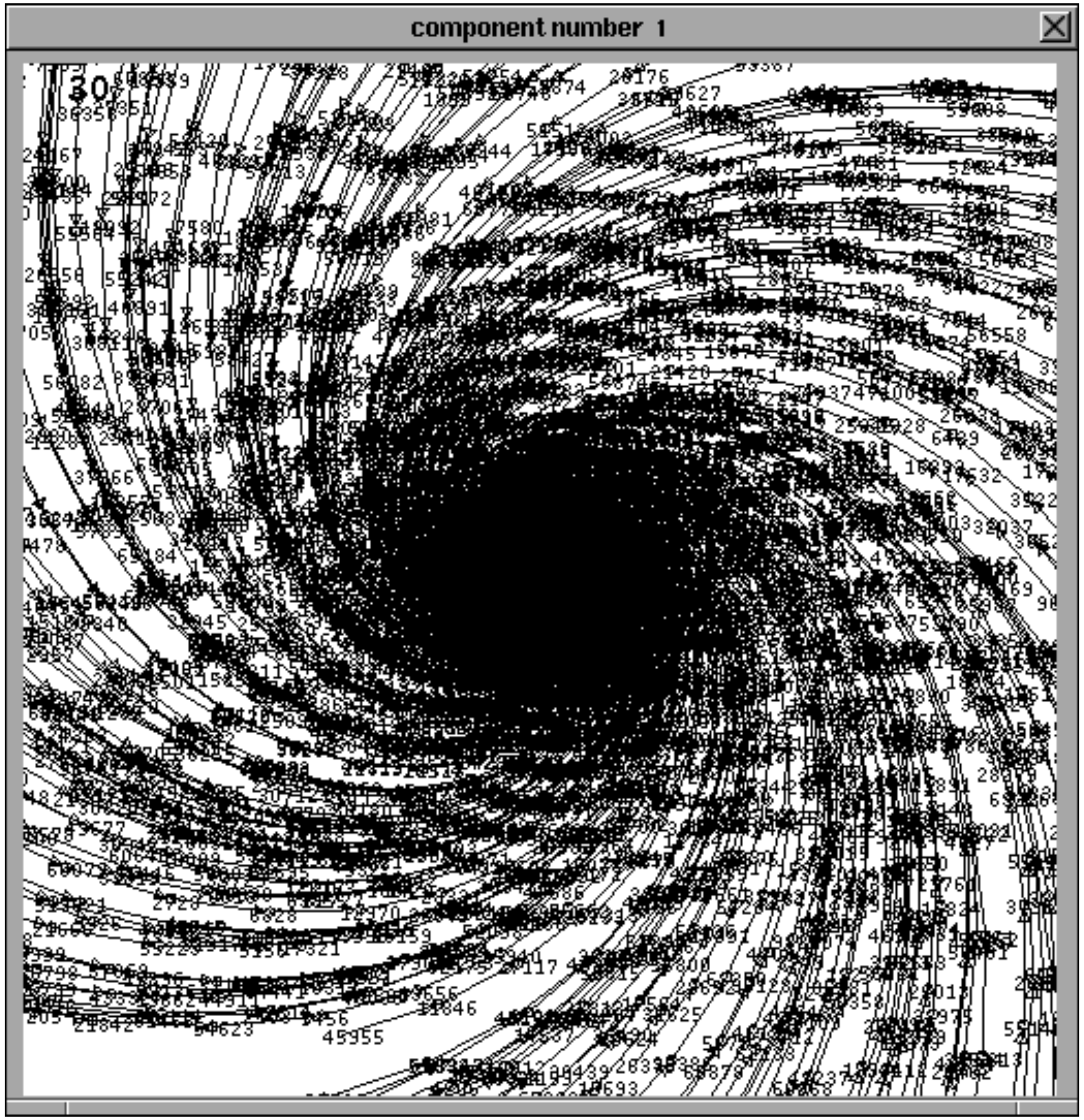
random initial condition 50%

Field of basin attractors: ECA rule 126

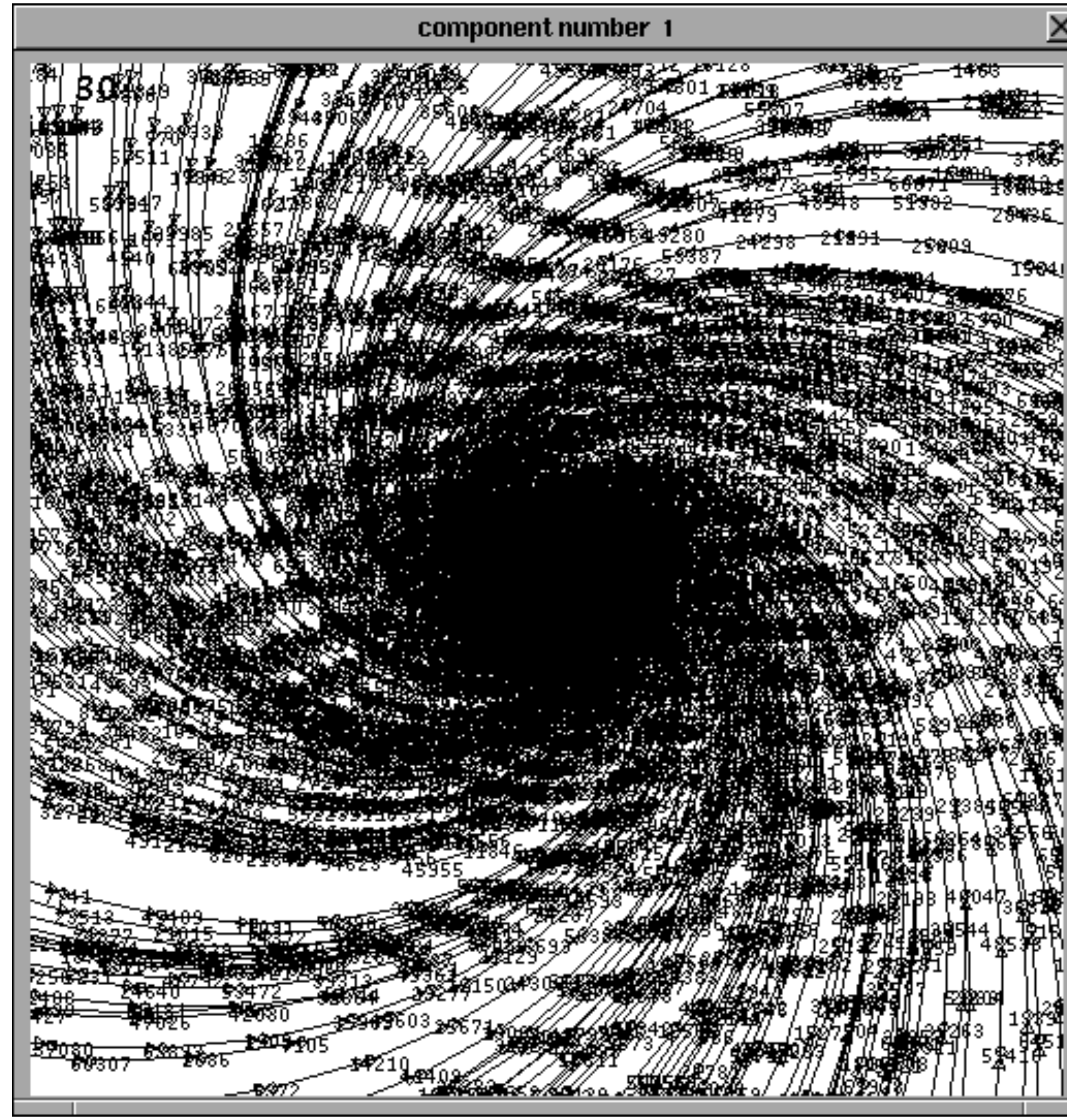


class III: very long transients, very long periodic attractors low in-degree, low leaf density (chaotic dynamics).

Field of basin attractors: ECA rule 30



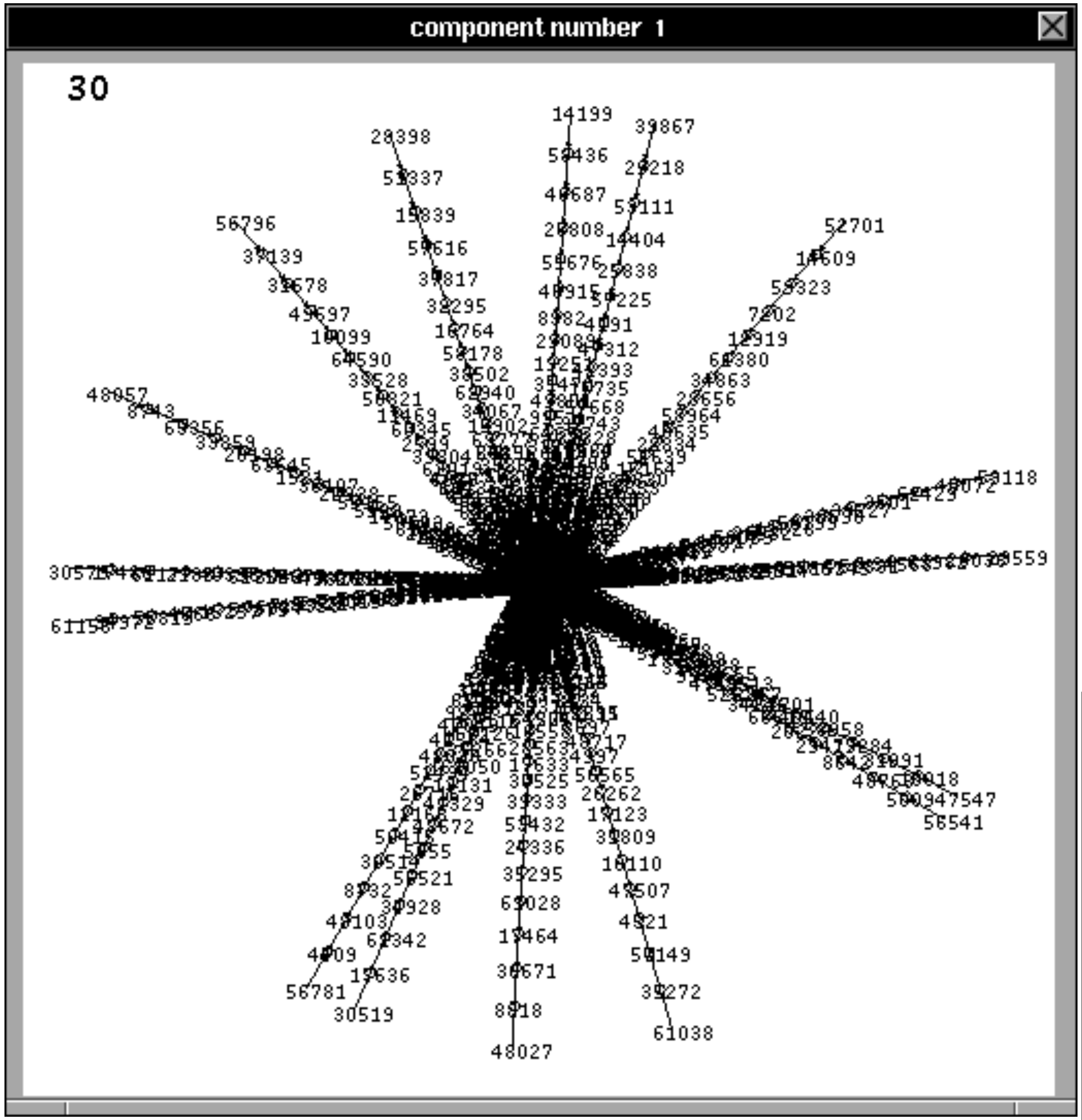
main attractor

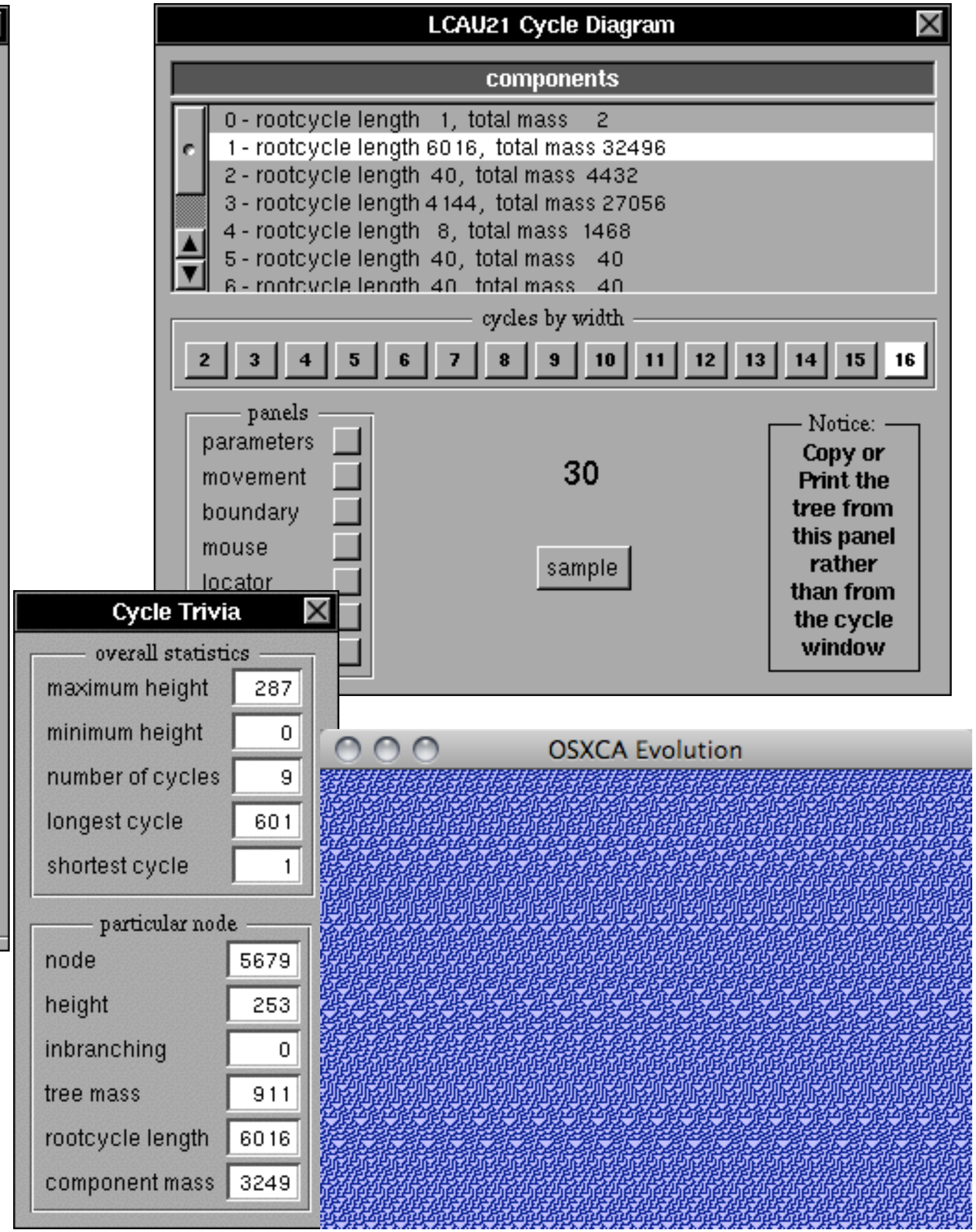


zoom out

H. V. McIntosh, ***NXLCAU systems***, http://delta.cs.cinvestav.mx/~mcintosh/oldweb/software.html

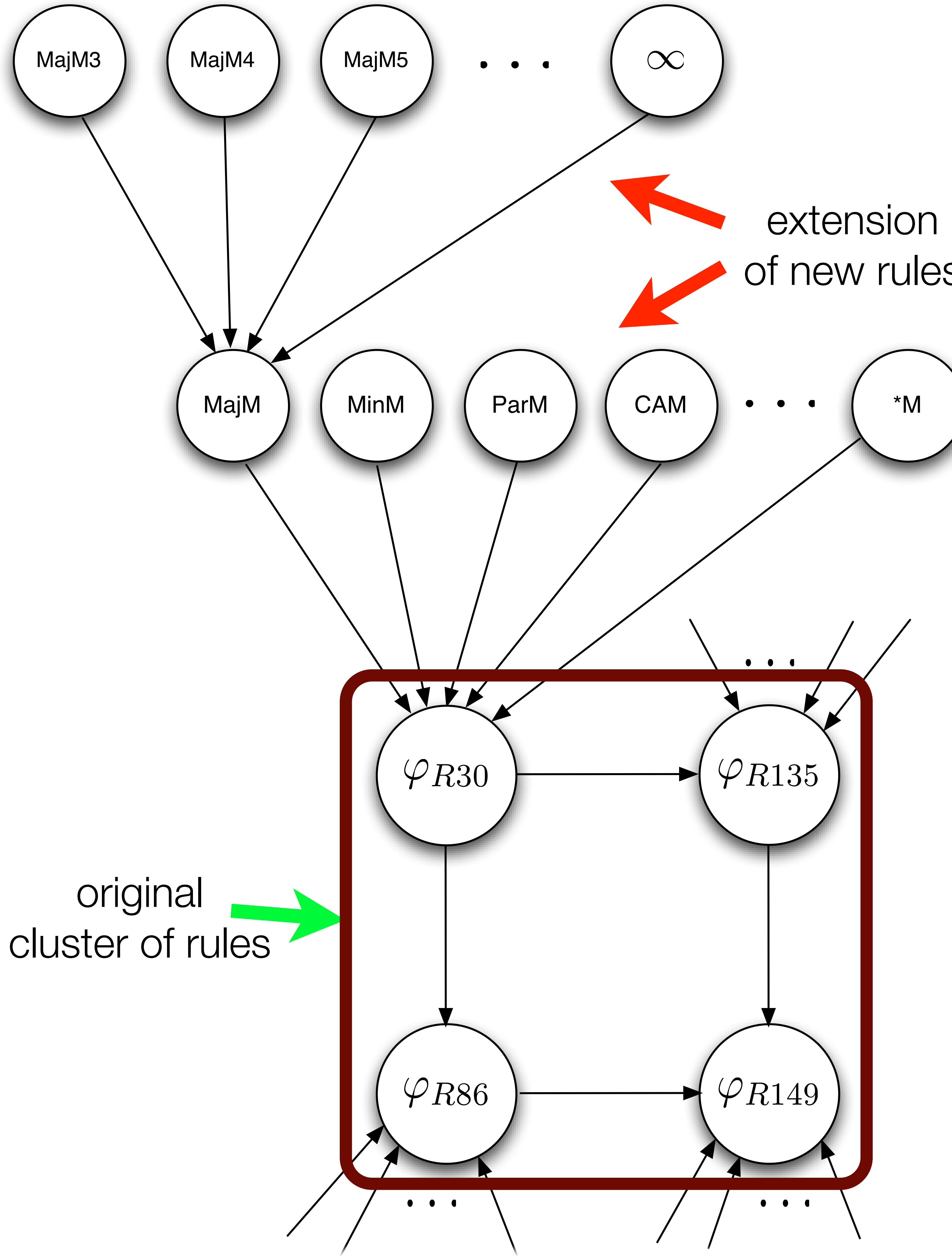
Field of basin attractors: ECA rule 30





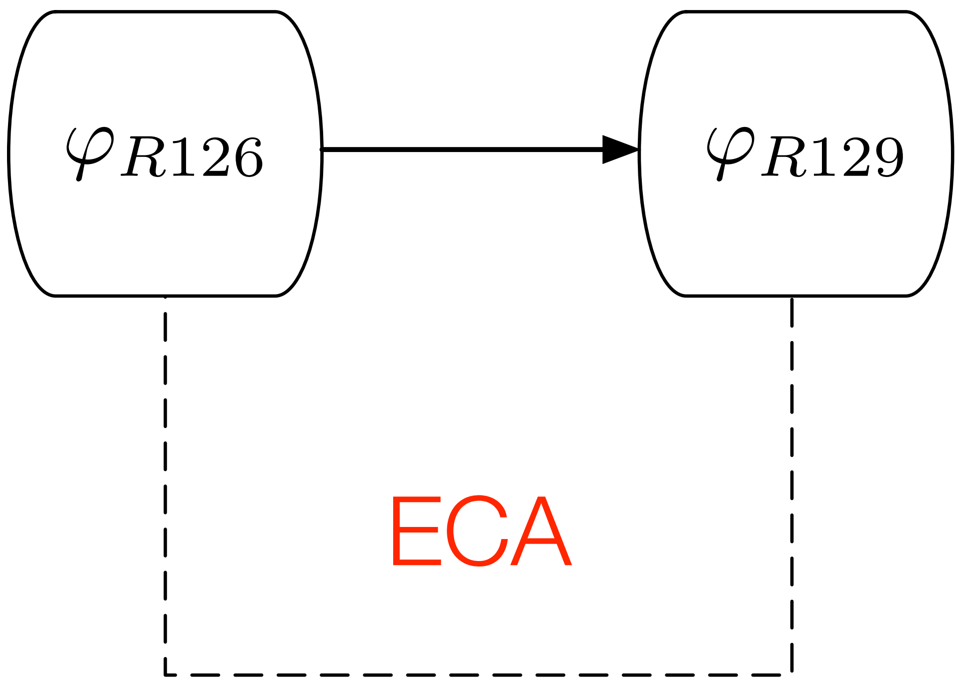
final zoom out

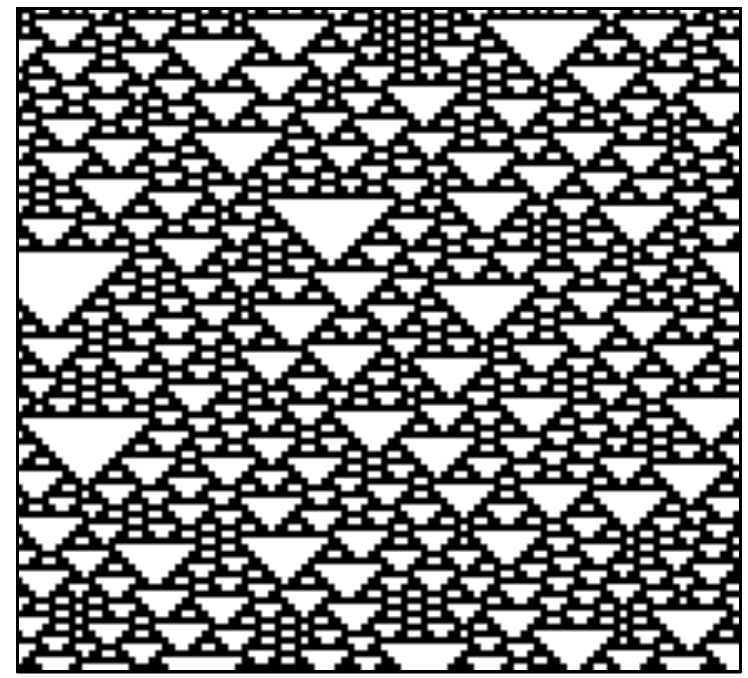
Expanding the ECA universe to new rules!

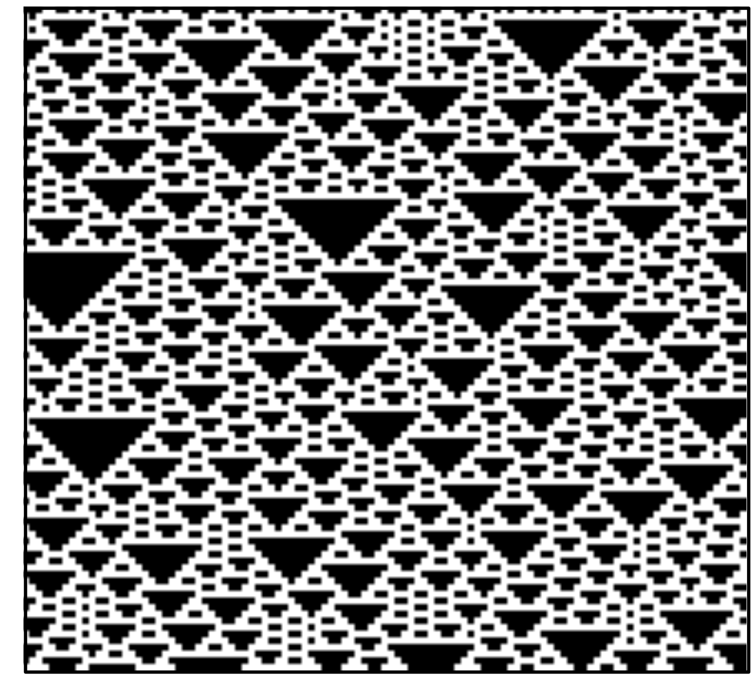


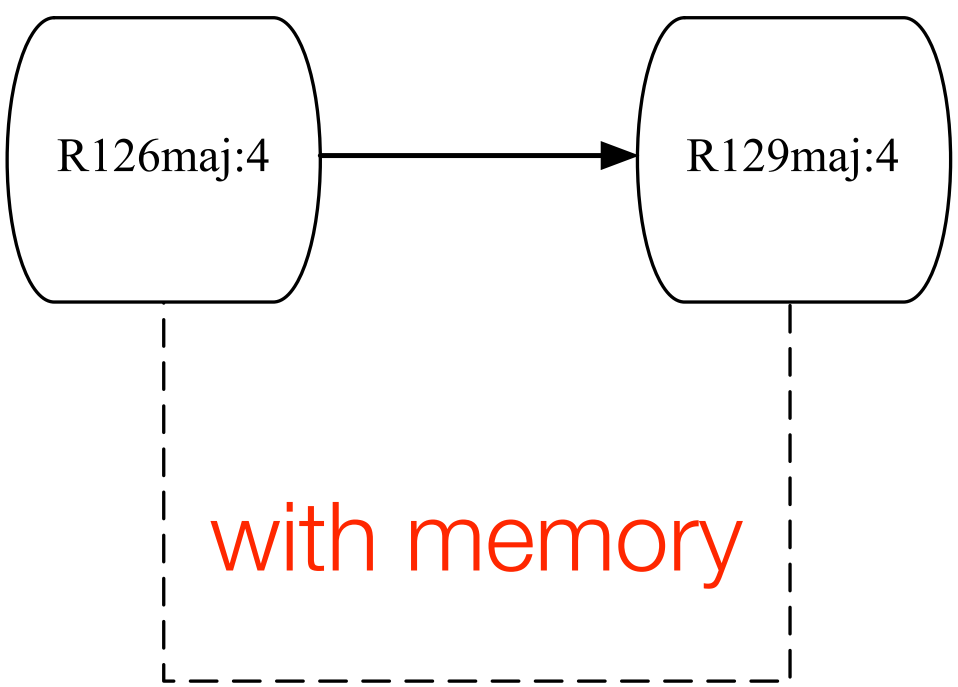
A. Wuensche & M. Lesser (1992) ***The Global Dynamics of Cellular Automata***, Addison-Wesley Publishing.

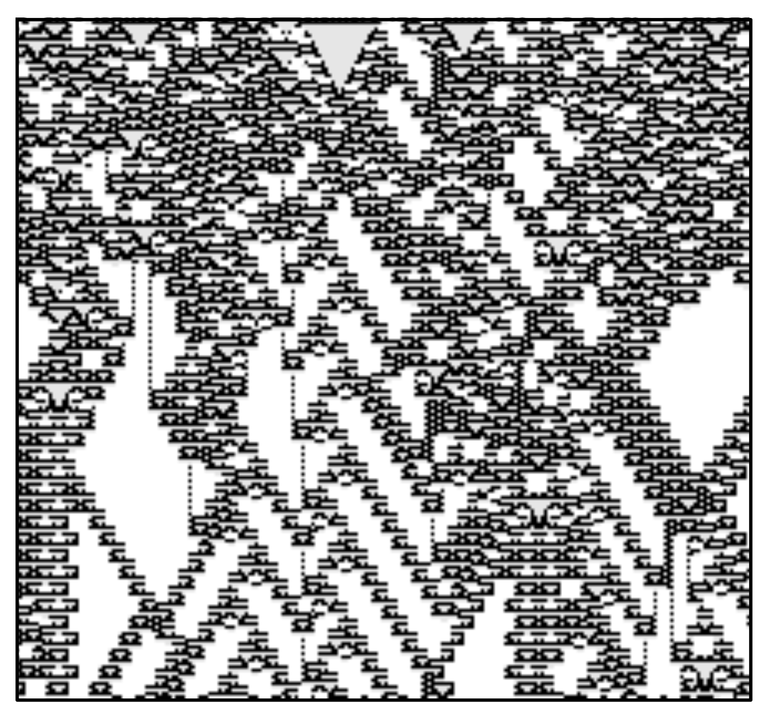
Cluster of equivalents rules for ECA rule 126 including memory function

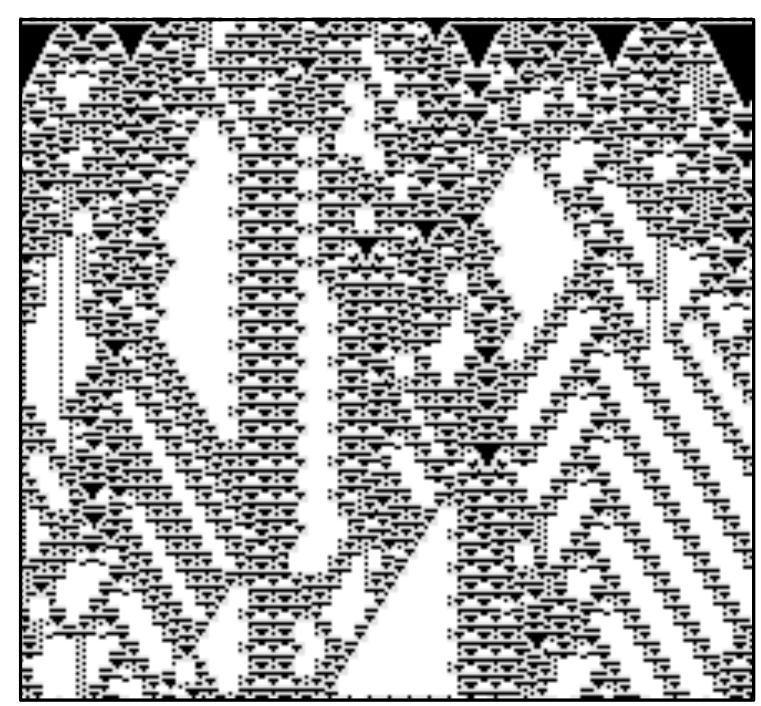




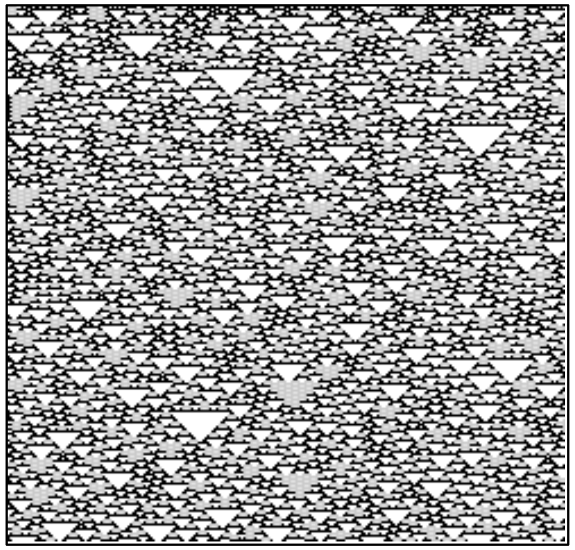


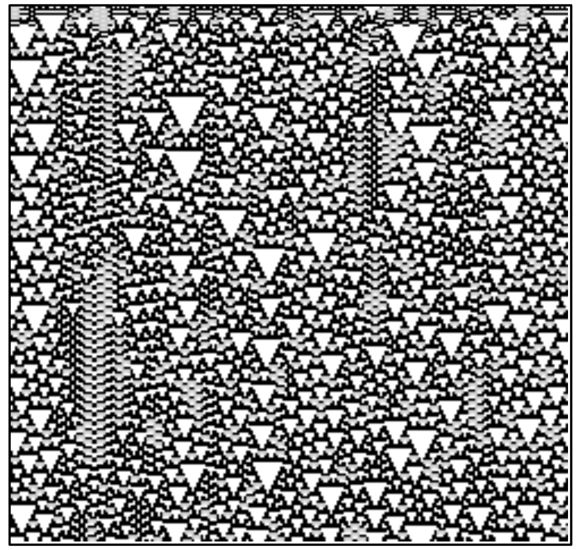


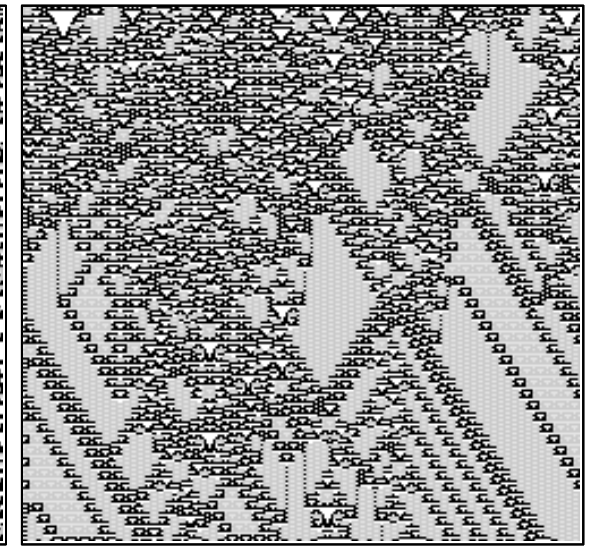


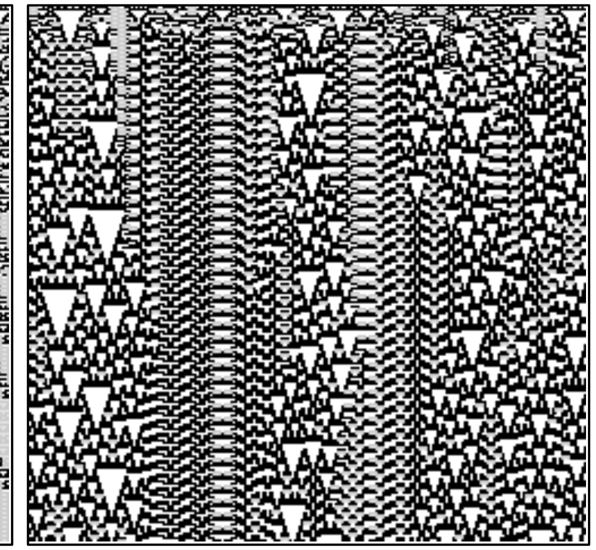


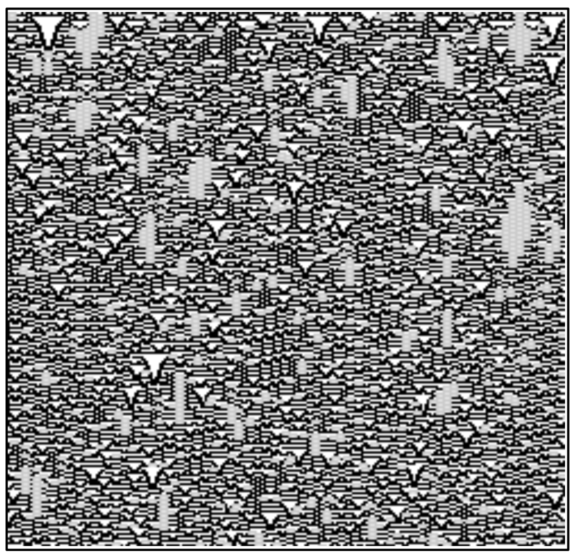
ECA rule 126 with majority memory

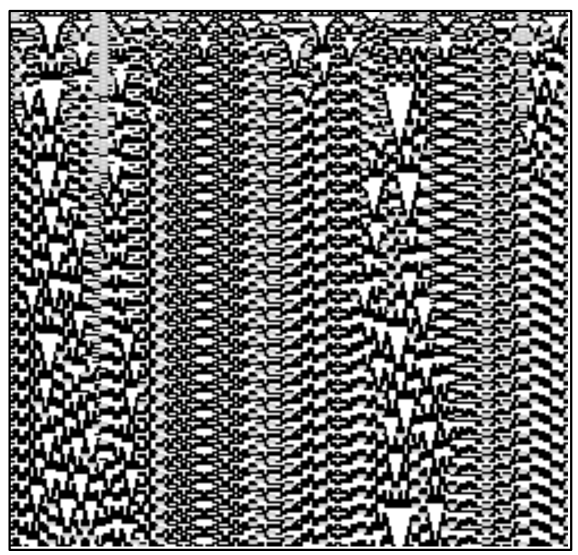

original

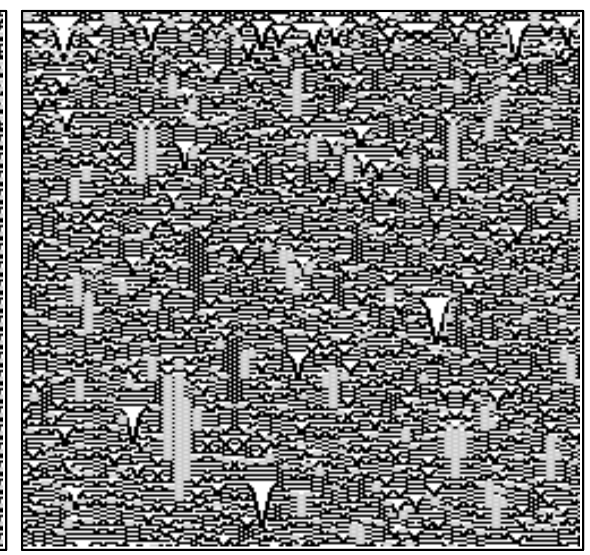

$\tau = 3$

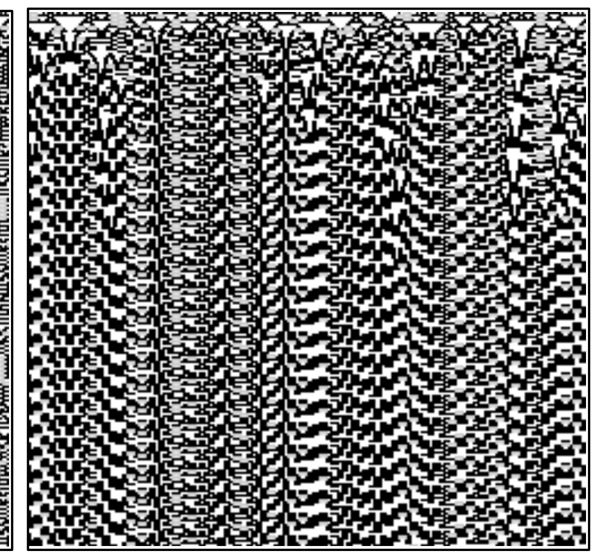

$\tau = 4$

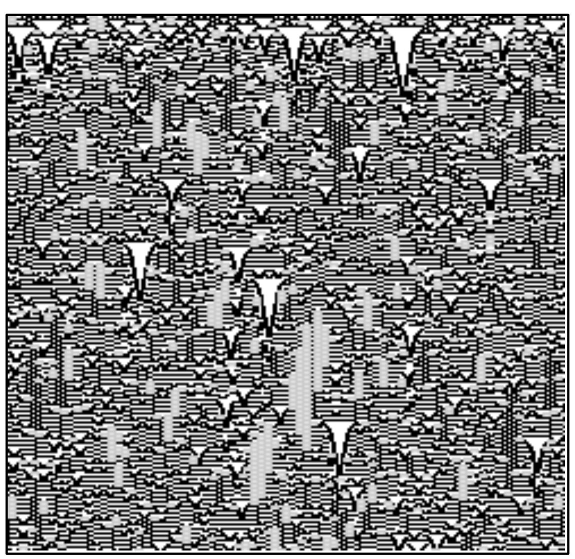

$\tau = 5$

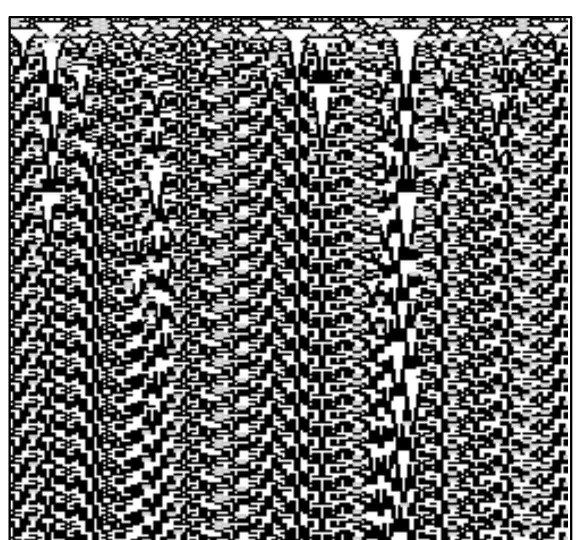

$\tau = 6$

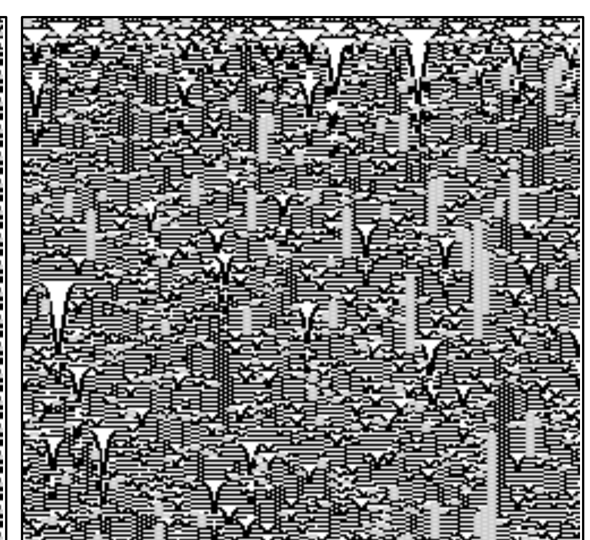

$\tau = 7$

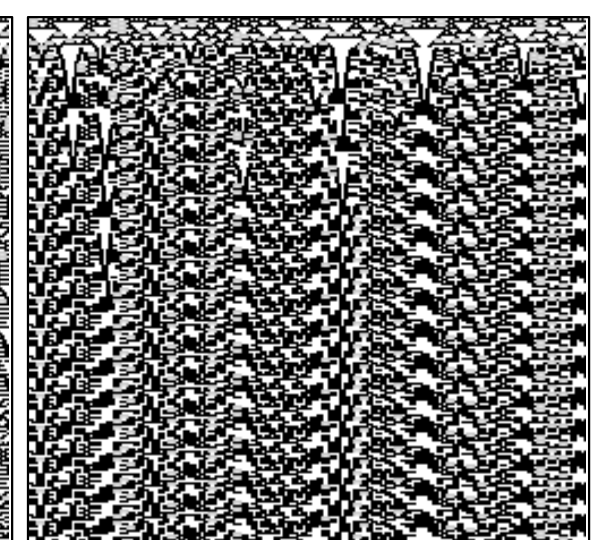

$\tau = 8$


$\tau = 9$


$\tau = 10$

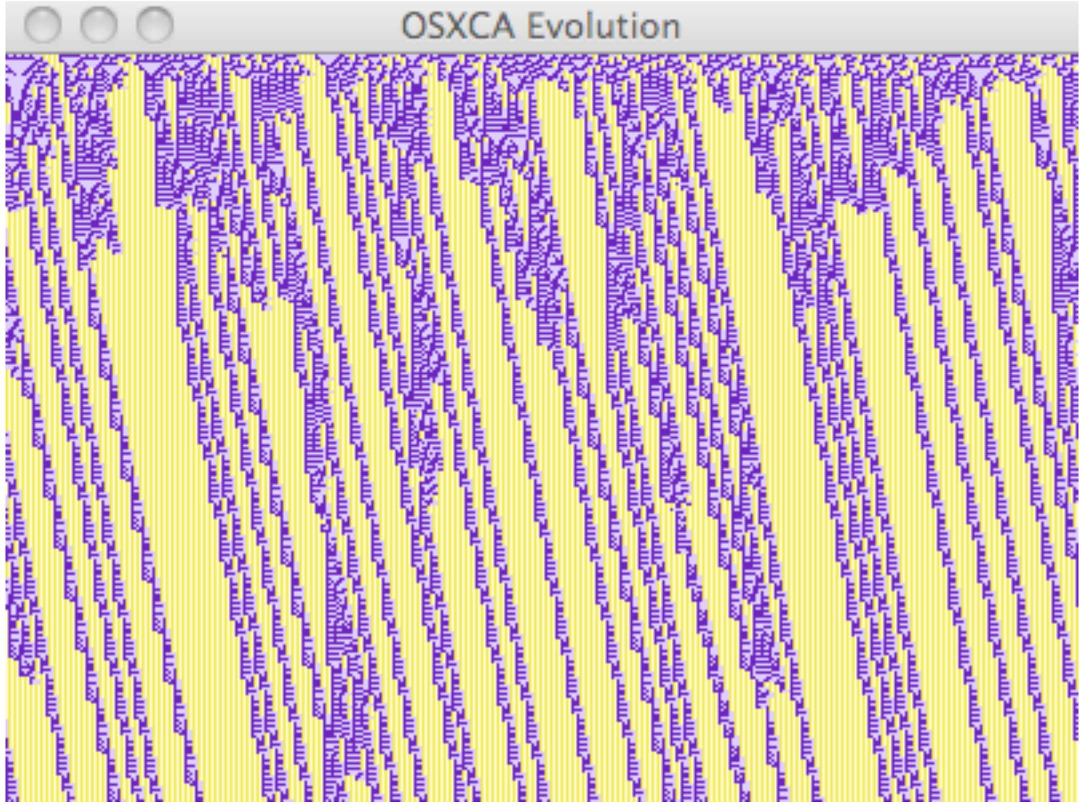

$\tau = 11$


$\tau = 12$

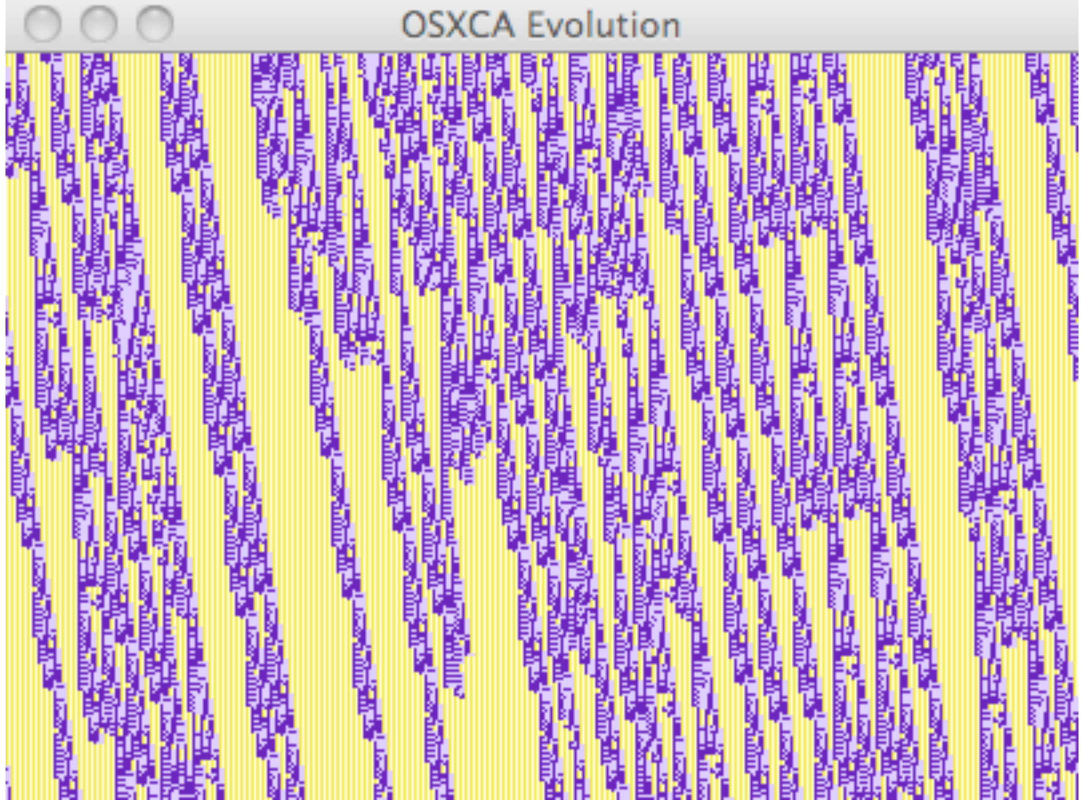

$\tau = 13$

ECA Rule 30 with memory (ECAM)

ECAM $\Phi_{R30maj:8}$



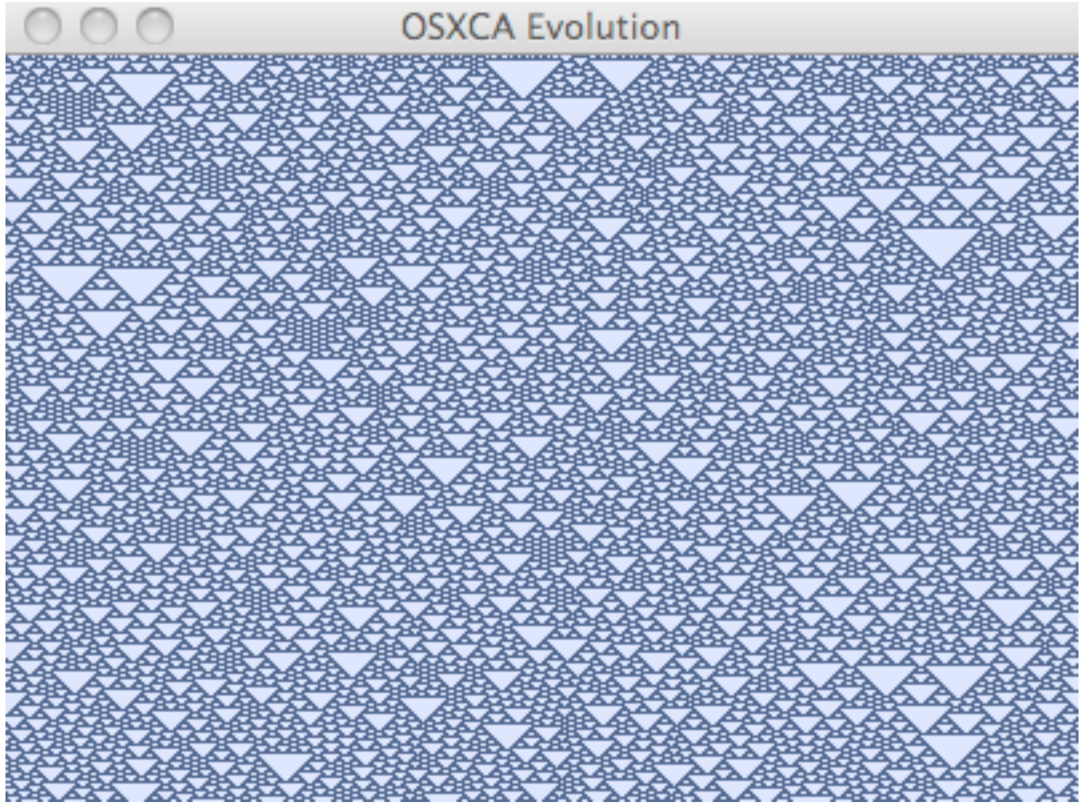
ECA Rule 30
ahistoric (conventional) evolution



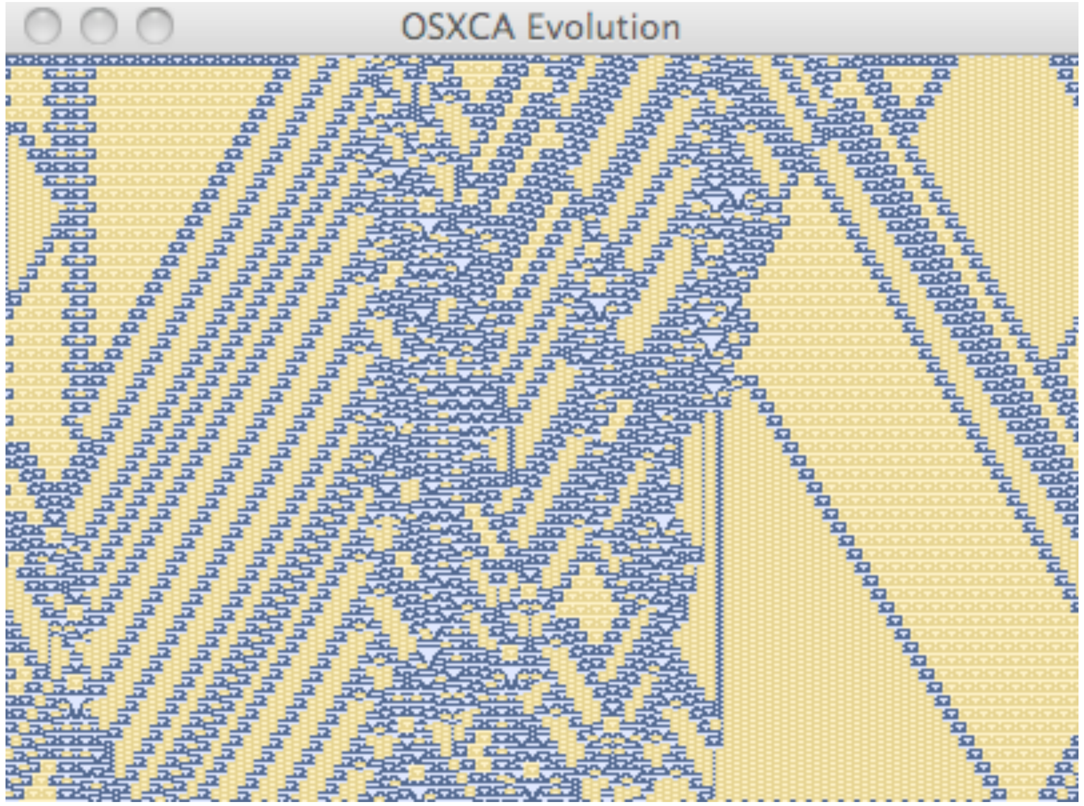
ECAM $\Phi_{R30maj:12}$

Genaro J. Martínez, Andrew Adamatzky, Ramon Alonso-Sanz, and Juan C. Seck-Tuoh-Mora, "**Complex dynamics emerging in Rule 30 with majority memory**", *Complex Systems* 18(3), 345-365, 2010.

ECA rule 126 with memory (ECAM)



ECA Rule 126
ahistoric (conventional) evolution



ECAM $\Phi_{R126maj:4}$

Genaro J. Martínez, Andrew Adamatzky, Juan C. Seck-Tuoh-Mora, and Ramon Alonso-Sanz, "**How to make dull cellular automata complex by adding memory: Rule 126 case study**", *Complexity* 15(6), 34-49, 2010.

starting with a single cell in state 1

CA classification

classification		
type	**num.**	**rules**
class I	8	0, 8, 32, 40, 128, 136, 160, 168.
class II	65	1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 19, 23, 24, 25, 26, 27, 28, 29, 33, 34, 35, 36, 37, 38, 42, 43, 44, 46, 50, 51, 56, 57, 58, 62, 72, 73, 74, 76, 77, 78, 94, 104, 108, 130, 132, 134, 138, 140, 142, 152, 154, 156, 162, 164, 170, 172, 178, 184, 200, 204, 232.
class III	11	18, 22, 30, 45, 60, 90, 105, 122, 126, 146, 150.
class IV	4	41, 54, 106, 110.

TABLE 2
Wolfram's classification relation.

The main interest of chaotic rules relate to developing cryptography, random number generators, and fields of attraction. However, the so called class IV or complex rules have captured most attention given their potential for computational universality, and their applications in artificial life by the simulations of particles, waves, mobile self-localizations, or gliders. Their capacity to contain intrinsically complex systems. This kind of discrepancy between chaotic rules, and complex rules capable of computational universality, are discussed in the CA literature.

Genaro J. Martínez, "**A Note on Elementary Cellular Automata Classification**", *Journal of Cellular Automata* 8(3-4), 233-259, 2013.

CA classification with memory

classification		
type	**num.**	**rules**
strong	39	2, 7, 9, 10, 11, 15, 18, 22, 24, 25, 26, 30, 34, 35, 41, 42, 45, 46, 54, 56, 57, 58, 62, 94, 106, 108, 110, 122, 126, 128, 130, 138, 146, 152, 154, 162, 170, 178, 184.
moderate	34	1, 3, 4, 5, 6, 8, 13, 14, 27, 28, 29, 32, 33, 37, 38, 40, 43, 44, 72, 73, 74, 77, 78, 104, 132, 134, 136, 140, 142, 156, 160, 164, 168, 172.
weak	15	0, 12, 19, 23, 36, 50, 51, 60, 76, 90, 105, 150, 200, 204, 232.

TABLE 4
ECAM's classification relation.

strong, because the memory functions are unable to transform one class to another;

moderate, because the memory function can transform the rule to another class and conserve the same class as well;

weak, because the memory functions do most transformations and the rule changes to another different class quickly.

CA classification with memory

3.2 Some relevant properties

Memory classification presents a number of interesting properties.

We have ECA rules which composed with a particular kind of memory are able of reach another class including the original dynamic. The main feature is that, at least, this rule with memory is able to reach every different class. Rules with this property are called *universal ECAM* (5 rules).

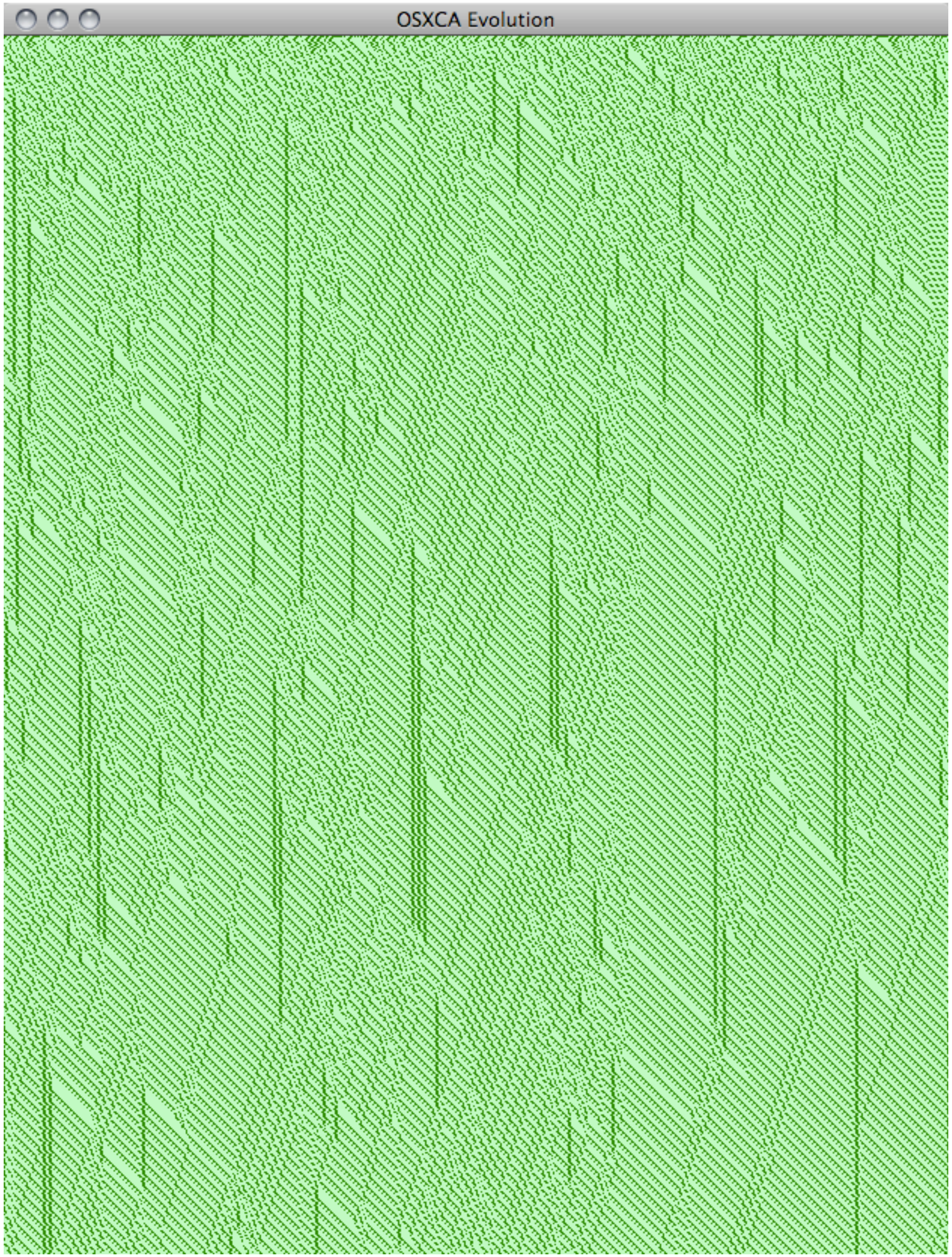
universal ECAM: 22, 54, 146, 130, 152.

On the other hand, we have ECA that when composed with memory are able to yield a complex ECAM but with elements of the original ECA rule. They are called *complex ECAM* (44 rules).

complex ECAM: 6, 9, 10, 11, 13, 15, 22, 24, 25, 26, 27, 30, 33, 35, 38, 40, 41, 42, 44, 46, 54, 57, 58, 62, 72, 77, 78, 106, 108, 110, 122, 126, 130, 132, 138, 142, 146, 152, 156, 162, 170, 172, 178, 184.

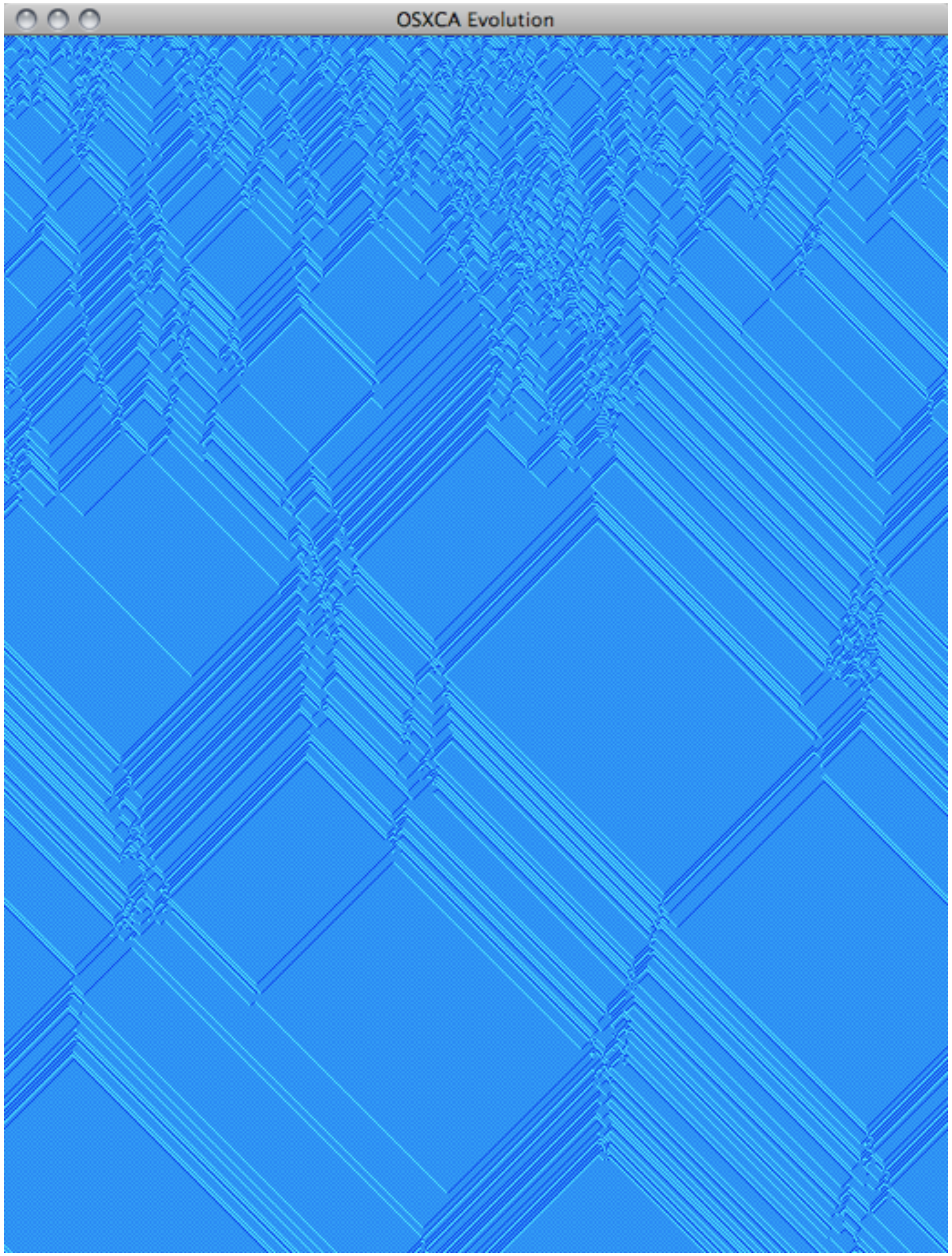
New complex ECAM evolution rules with memory

$\phi_{R10par:3}$



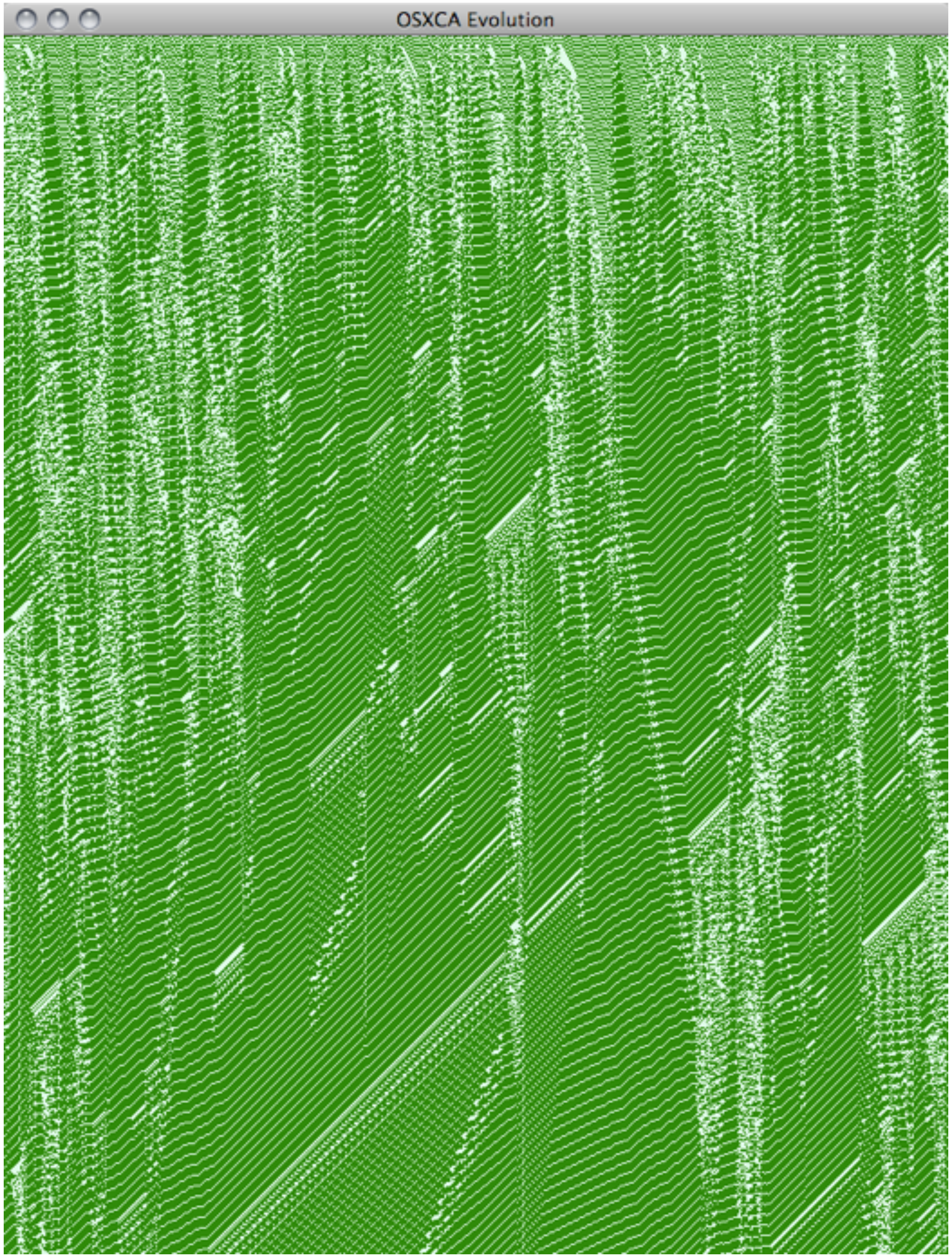
New complex ECAM evolution rules with memory

$\phi_{R57maj:8}$

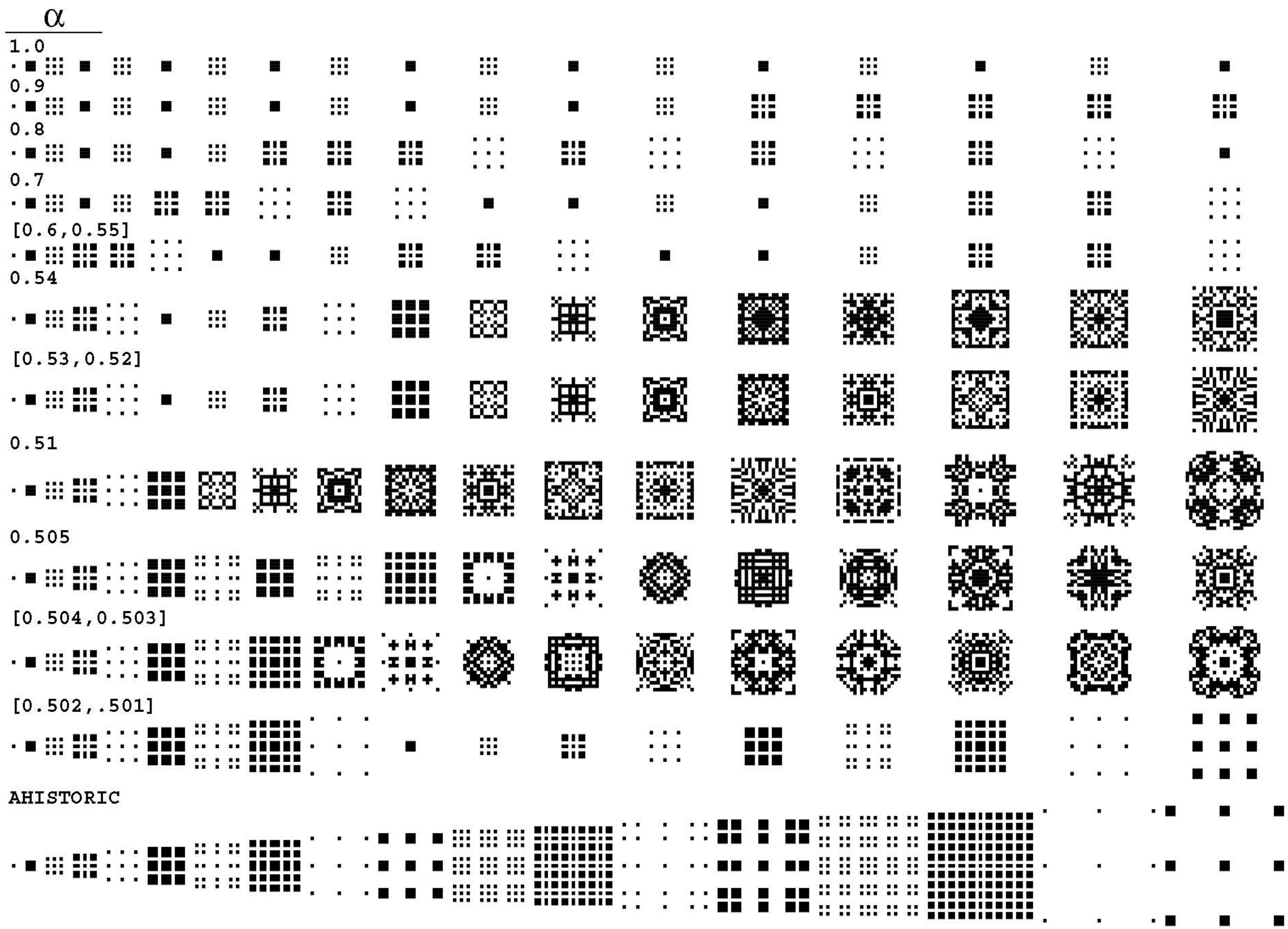


New complex ECAM evolution rules with memory

$\phi_{R27par:6}$



The 2D PARITY rule with Memory. Moore N. [17]



Alonso-Sanz,R.,Martin,M.(2002). Cellular Automata with Memory: patterns starting with a single site seed. *IJMPC*,13,1.

Of course, we can select memory function in 2D, 3D, etc. (slide thanks to Alonso-Sanz, 2012)

A novel of cellular automata evolution rules emerge selecting a kind of memory. So, these set of rules as conventional cellular automata can find potential applications in:

- unconventional computation
- physics (solitons, particle collisions)
- mathematics
- biological phenomena
- chemical reactions
- reaction-diffusion systems
- simulation of populations, societies, and virus
- complex systems, artificial life, chaos, and fractals

Conclusions

We can conclude that information on some dynamical system can be found on any *class*, selecting a kind of memory for discover it.

Selecting different kinds of memories for a specific CA, we can proof experimentally that its behaviour can change to any other possible class including itself. This way, determines if a CA belong to a respective class or not, match with another previous results founded for other researchers. Such that, CA classification is a undecidable problem.

The End

Thank you very much for you kind attention!

questions?

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